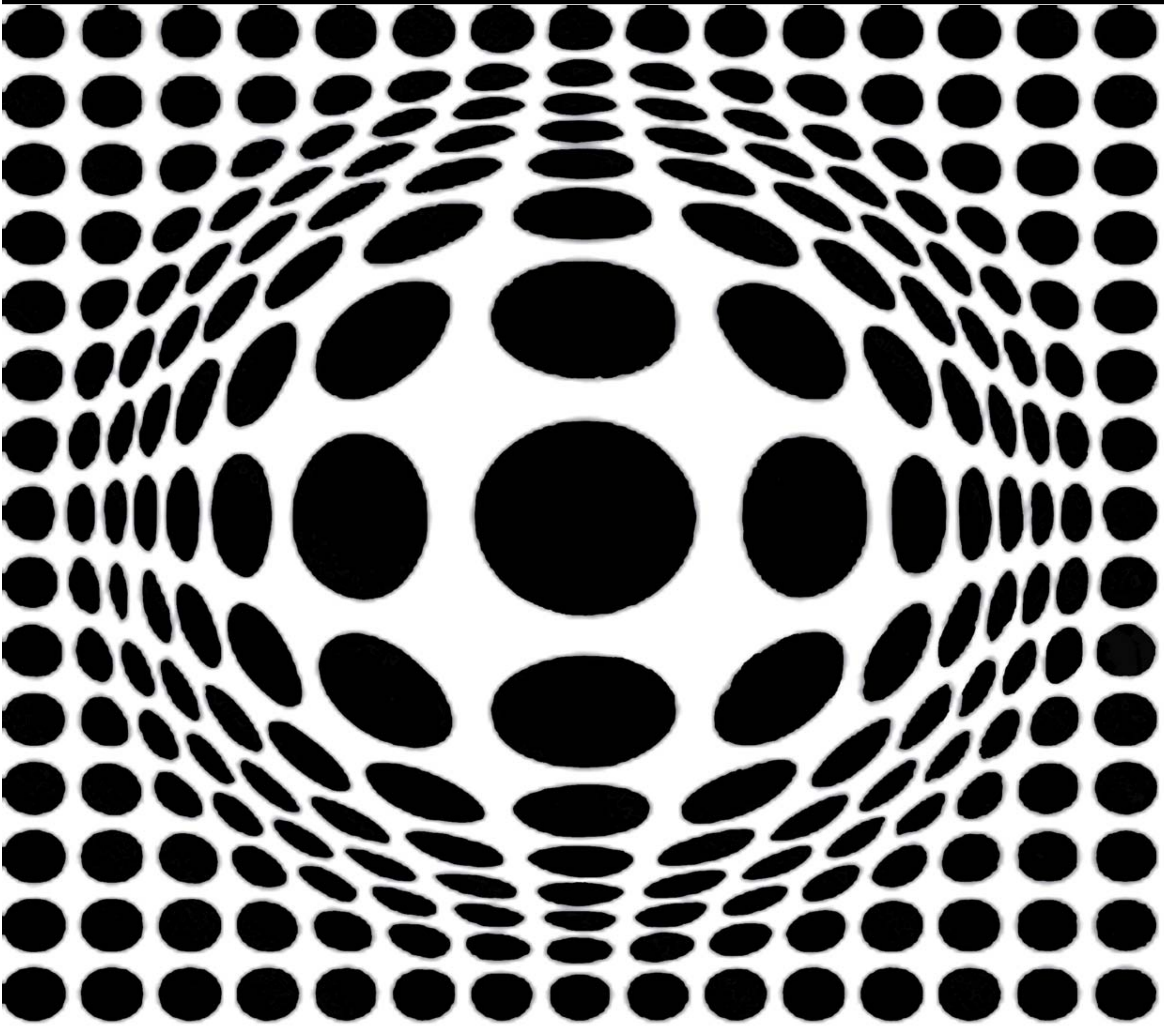
# Maths Module 4: Geometry

## Student's Book



The Curriculum Project

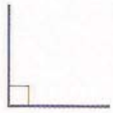
# Maths Module 4: Geometry and Trigonometry

# Contents

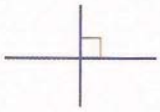
# 1. Shapes

## 1.1 Angles

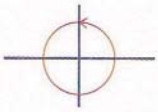
You will probably recognise some of the following types of angles:



A quarter turn is called a **right angle**.

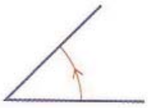


Two lines at right angles are **perpendicular**.

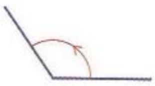


A full turn is 360°.

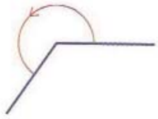
The angle on a straight line is 180°.



An angle between 0° and 90° is **acute**.

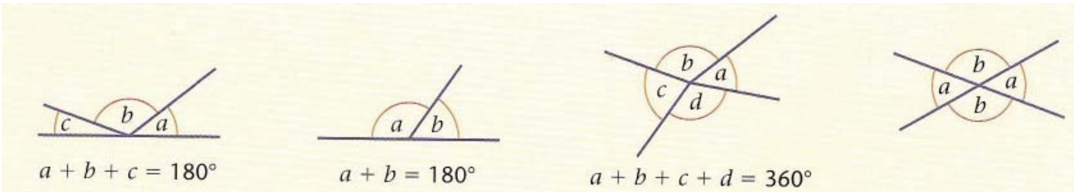


An angle between 90° and 180° is **obtuse**.



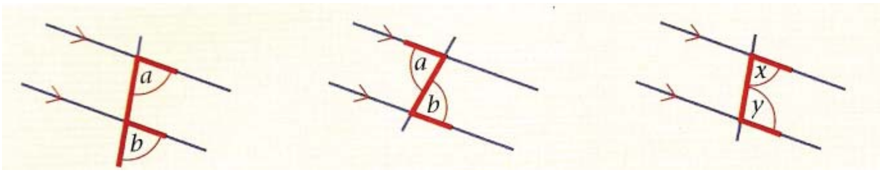
An angle between 180° and 360° is a **reflex** angle.

These properties of angles are also useful:




A pair of angles which sum to 180° are called **supplementary angles**.

Two straight lines which cross at a point have opposite angles which are equal.

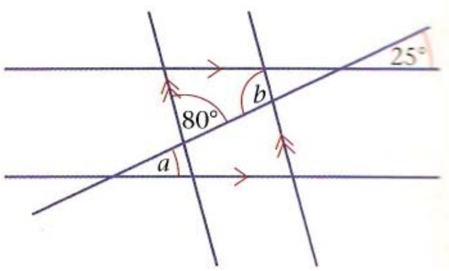



**Corresponding angles** are equal $a = b$.

**Alternate angles** are equal $a = b$.

**Co-interior angles** are supplementary $x + y = 180°$.

**Example -** Find angles $a$ and $b$, giving reasons for your answer.




$a$ and 25° are corresponding angles, so $a = 25°$.

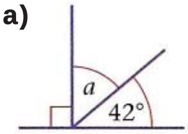
80° and $b$ are co-interior angles, so:

$80° + b = 180°$, $b = 180° - 80° = 100°$.

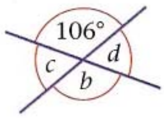
**Practice**

**i.** Calculate the size of each lettered angle

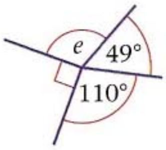
**a)**



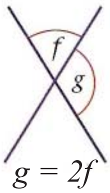

**b)**



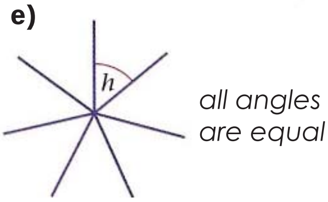

**c)**




**d)**

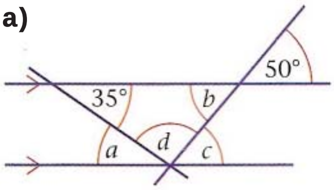



**e)**

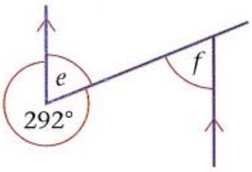



**ii.** Calculate each of the lettered angles

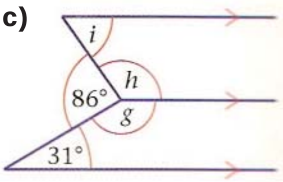
**a)**



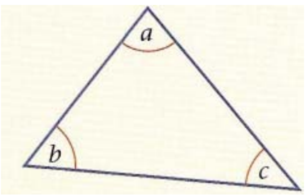

**b)**



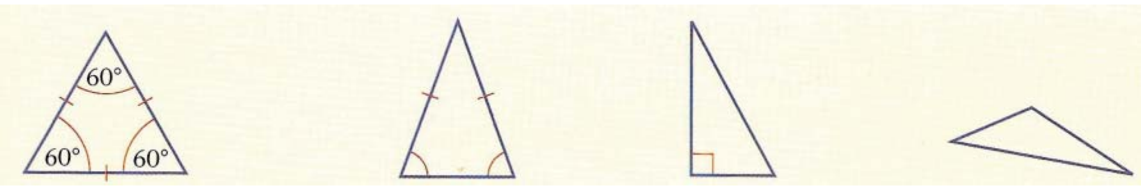

**c)**




## 1.2 Triangles




The interior angles of a **triangle** always add up to 180°.
$a + b + c = 180°$.



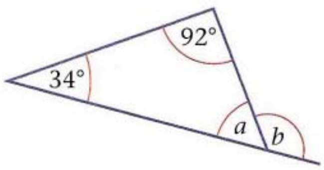

An **equilateral triangle** has:
- 3 sides equal
- 3 angles equal (60°).

An **isoceles triangle** has:
- 2 sides equal
- base angles are equal.

A **right-angled triangle** has:
- 1 angle of 90°.

Triangles with none of these properties are called **scalene triangles**.

**Example -** Calculate the sizes of the lettered angles, giving reasons for your answers.



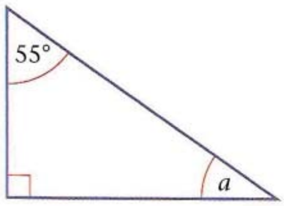

$34° + 92° + a = 180°$ (interior angles of a triangle)
$a = 180° - 126° = 54°$

$a + b = 180°$ (angles on a straight line)
$b = 180° - 54° = 126°$

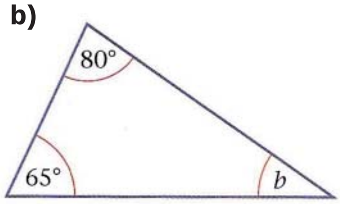
**Practice**

Calculate the size of each lettered angle. Give reasons for your answer.

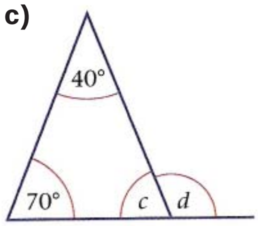
**a)**



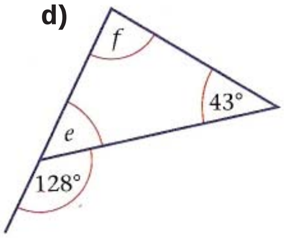

**b)**



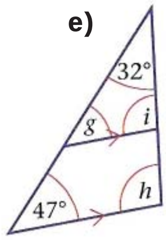

**c)**




**d)**

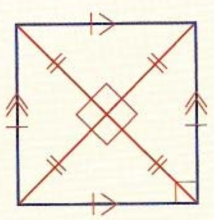



**e)**

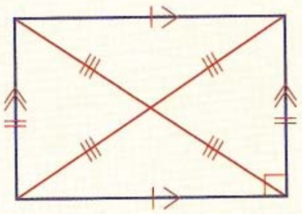



## 1.3 Quadrilaterals

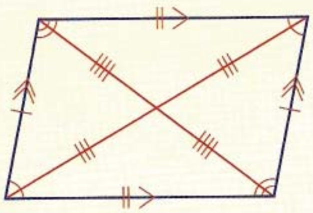
A **quadrilateral** is a four sided shape. Some of them have special properties.



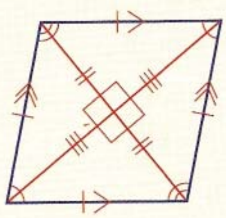
A **square** has:
- all sides equal
- opposite sides **parallel**
- all interior angles 90°.



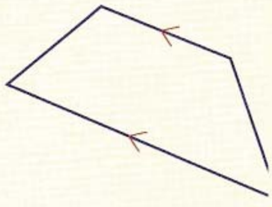
A **rectangle** has:
- opposite sides equal
- opposite sides parallel
- all interior angles 90°
- diagonals that bisect each other.



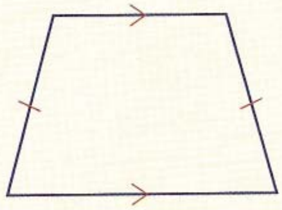
A **parallelogram** has:
- opposite sides equal
- opposite sides parallel
- diagonally opposite angles equal
- adjacent angles are supplementary.



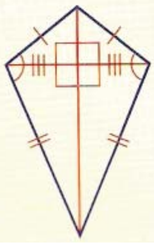
A **rhombus** has:
- all sides equal
- opposite sides parallel
- opposite angles equal
- adjacent angles are supplementary.



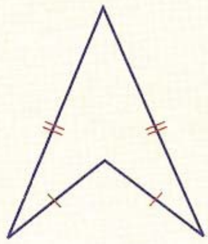
A **trapezium** has:
- 1 pair of parallel lines.



An isoceles **trapezium** has:
- 1 pair of parallel lines
- 2 sides equal.



A **kite** has:
- 2 pairs of adjacent sides equal
- 1 pair of opposite angles equal
- **diagonals** cut at 90°.

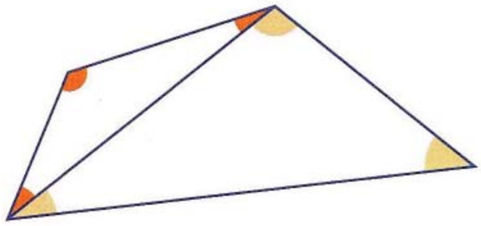


An **arrowhead** has:
- 2 pairs of adjacent sides equal.

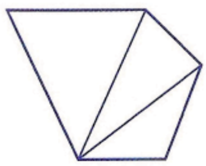
## 1.4 Interior Angles

The interior angles of a quadrilateral add up to 360°. You can see this by dividing it into two triangles, as shown in the diagram.

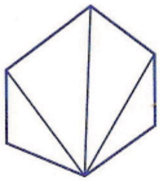
The angles in a triangle sum to 180°. There are two triangles in the quadrilateral, so the angles sum to 360°.



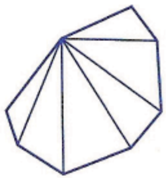
The sum of the interior angles of any shape can be found by dividing the shape into triangles from one vertex.



**Pentagon**
3 triangles
Sum of interior angles = 3 x 180°
= 540°



**Hexagon**
4 triangles
Sum of interior angles = 4 x 180°
= 720°



**Octagon**
6 triangles
Sum of interior angles = 6 x 180°
= 1080°

Generally, the number of sides is 2 less than the number of sides of the **polygon**. So, for a polygon with $n$ sides: sum of the interior angles = $(n - 2) \times 180°$.

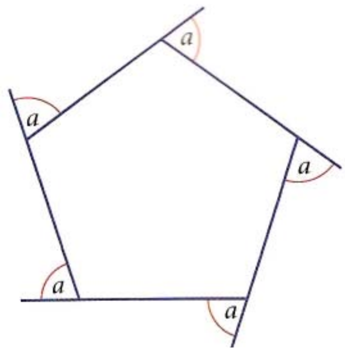
A **regular polygon** has all sides and angles equal. If a polygon is regular each interior angle can be calculated from:

$$\text{interior angle} = [(n - 2) \times 180]/n$$

Sometimes it is easier to calculate the exterior angle.

The sum of the exterior angles of any polygon is 360°

So for a regular polygon with $n$ sides: exterior angle = $360°/n$ and interior angle = 180° - **exterior angle**.




$a + a + a + a + a = 360°$

### Practice

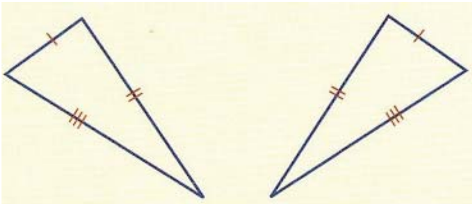
**i.** Write down the names given to these shapes:

**a)** A triangle with two sides equal
**b)** A quadrilateral with opposite sides equal
**c)** A quadrilateral with one pair of opposite sides parallel and equal
**d)** A quadrilateral with diagonals equal and intersecting at 90°

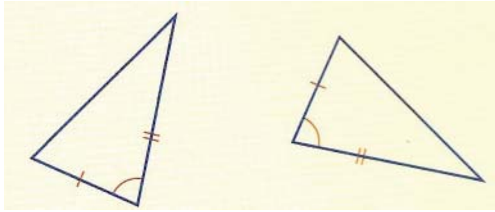
**ii.**
**a)** Work out the sum of the interior angles of a ten-sided shape (decagon)
**b)** Work out the interior angle of a regular decagon

## 1.5 Congruence

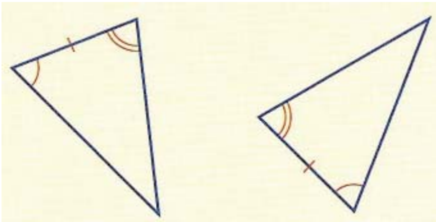
When 2-D shapes are exactly the same shape and size they are **congruent**. Triangles are congruent if:



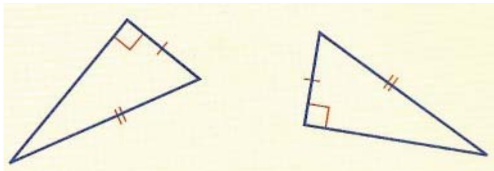
Three pairs of sides are equal.



Two pairs of sides are equal and the angles between them are equal.



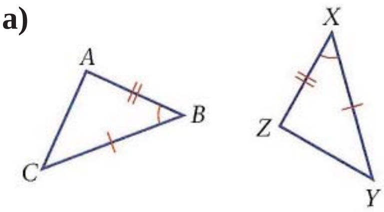
Two pairs of angles are equal and the sides between them are equal.



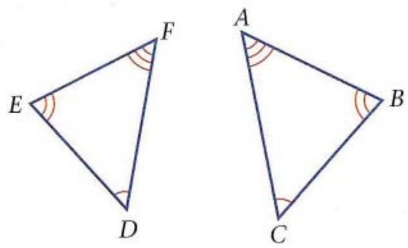
Both triangles have a right angle, the **hypotenuses** are equal and one pair of corresponding angles is equal.

**Example -** State whether these pairs of shapes are congruent. List the **vertices** in corresponding order and give reasons for congruency.

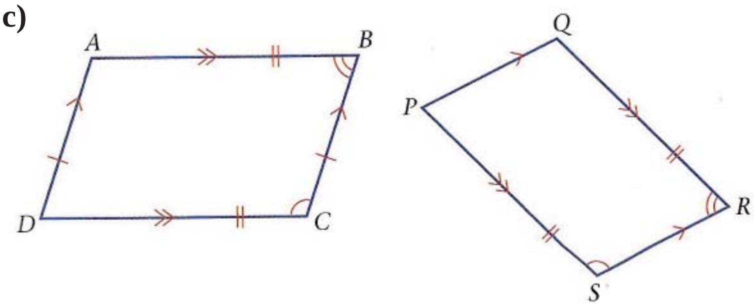
**a)**




**b)**




**c)**




**a)** Yes. *ABC* is congruent to *ZXY*.

**b)** No. Only the angles are equal. The corresponding sides may not be the same length.

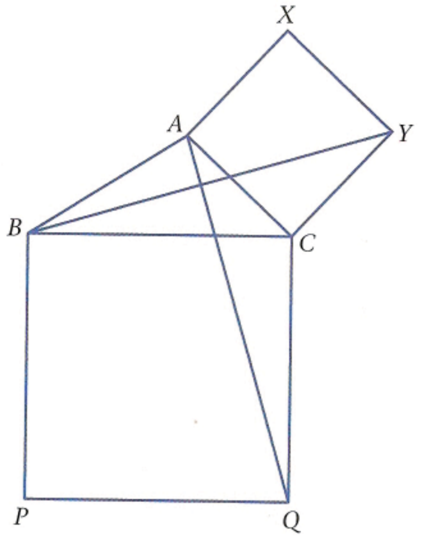
**c)** No. Parallelogram *ABCD* is not congruent with *QRSP*. It is not clear if *AD* = *PQ* or *BC* = *RS*.

**Example -** In the diagram, ACYX and BCQP are squares.
Prove that ACQ = BCY are congruent.

AC = CY
CQ = BC
∟ACQ = 90º + ∟ACB
∟BCY = 90º + ∟ACB

So, ∟ACQ = ∟BCY

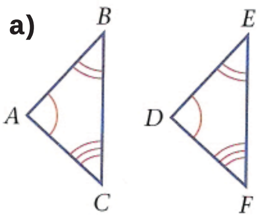
So triangles ACQ and YCB are congruent because two pairs of sides are equal and the angles between them are equal.



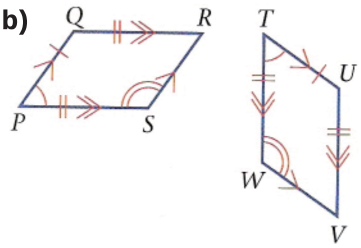

## Practice

**i.** Decide if the following pairs of shapes are congruent. Give reasons for your answer.

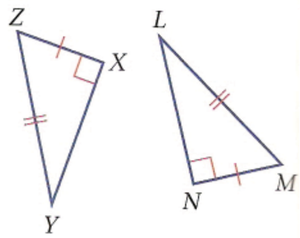
**a)**




**b)**

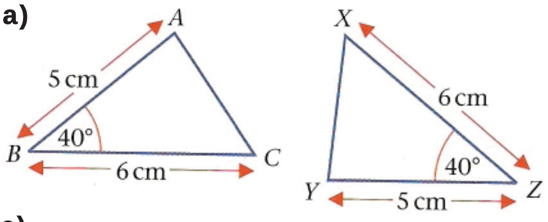



**c)**

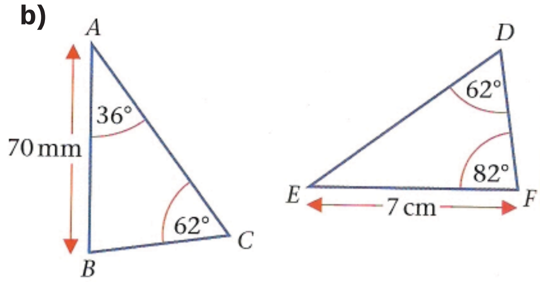



**ii.** Decide if the following pairs of shapes are congruent. Give reasons for your answer.

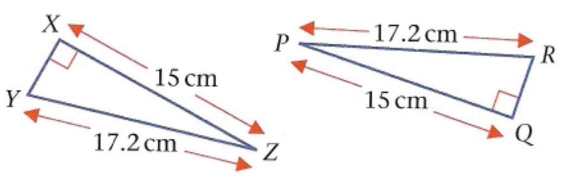
**a)**



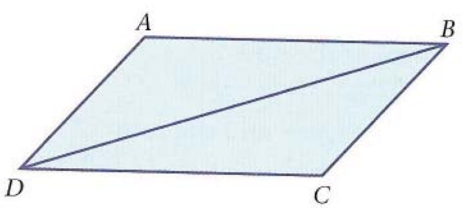

**b)**



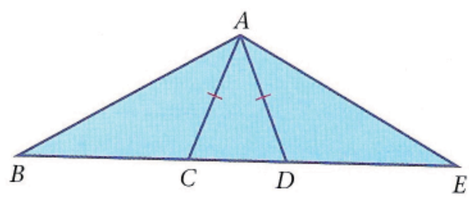

**c)**



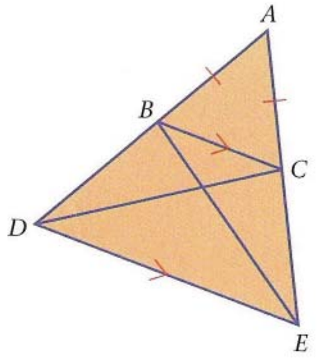

**iii.** ABCD is a parallelogram. Prove that ABD is congruent to CDB.



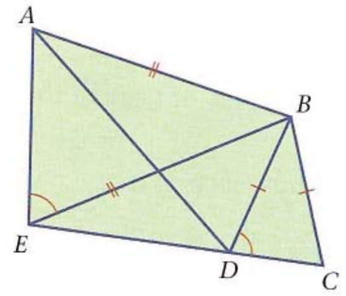

**iv.** In the diagram AC = AD and BD = CE. Prove that triangles ABC and ADE are congruent.



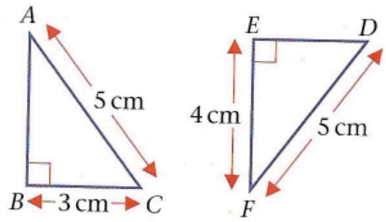
**v.** In the diagram, ABC is an isosceles triangle with AB = AC. Prove that triangles ACD and ABE are congruent.




**vi.** In the diagram AB = BE, BD = BC and angle AEB = angle BDC. Prove that triangles ABD and EBC are congruent.




**vii.** State whether the two triangles are congruent. Give reasons for your answers.




## 1.6 Similar Shapes

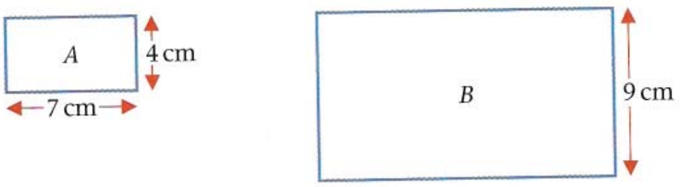
Shapes are similar if one shape is an **enlargement** of the other.

Polygons are similar if all corresponding angles are equal and the ratio of object length to image length is the same for all sides. The **scale factor** of an enlargement is the ratio:

$$\frac{\text{length of a side of one shape}}{\text{length of corresponding side on the other shape}}$$

**Example -** These rectangles are similar. *B* is an enlargement of *A*.

**a)** Find the scale factor of the enlargement
**b)** Find the missing length on rectangle *B*.

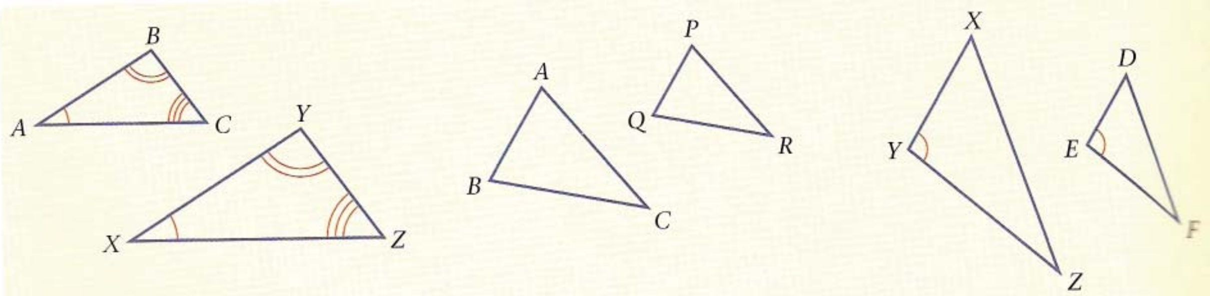



**a)** Using corresponding sides, the scale factor of the enlargement is 9/4 = 2.25

**b)** So the length of *B* is 7 x 2.25 = 15.75 cm

To decide whether two triangles are similar, you need to check that all the corresponding angles are equal, or that all the corresponding sides are in the same ratio.

Triangles are similar if one of these facts is true:




All corresponding angles are equal:

Angle A = Angle X
Angle B = Angle Y
Angle C = Angle Z

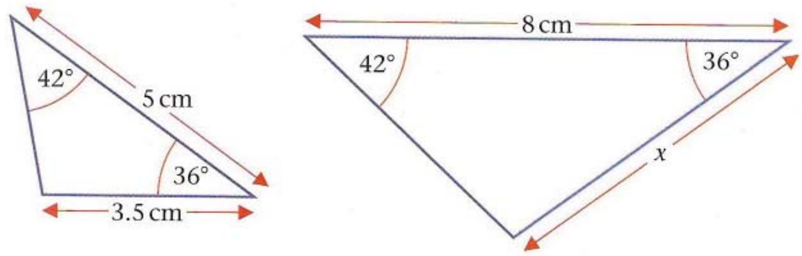
All corresponding sides are in the same ratio:

$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \text{Scale factor}$$

Two pairs of corresponding sides are in the same ratio and the included angles are equal:

$$\frac{XY}{DE} = \frac{YZ}{EF} \qquad \hat{Y} = \hat{E}$$

**Example -** Find the length of the side marked *x*.



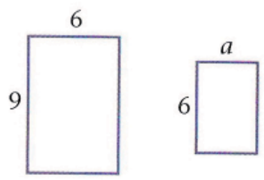

Two pairs of angles are equal, so the third pair must be equal. The triangles are similar. The scale factor of the enlargement is 8/5 = 1.6.

So, $x$ = 3.5 x 1.6 = 5.6 cm

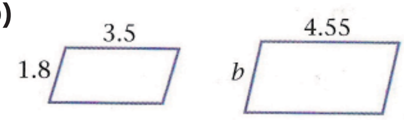
## Practice

**i.** Each pair of shapes is similar. Calculate each length marked by a letter.

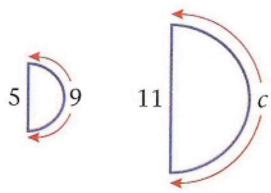
**a)**



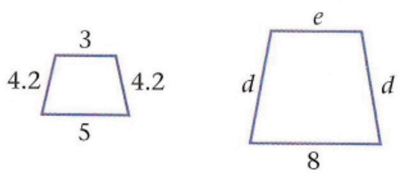

**b)**




**c)**

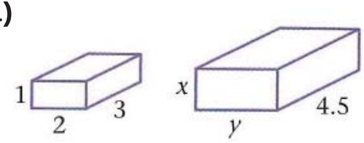



**d)**

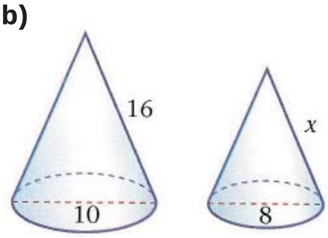


**ii.** Each pair of shapes is similar. Calculate the lengths marked by letters.

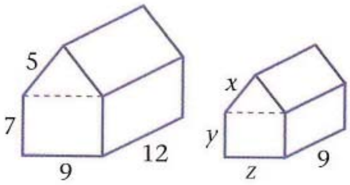
**a)**



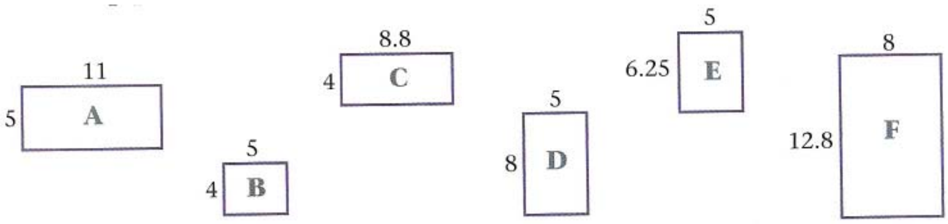

**b)**




**c)**

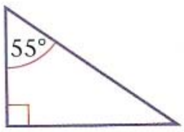



**iii.** Match the pairs of similar rectangles.

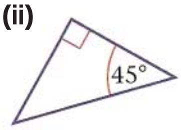



**iv.** Each group of 3 triangles has two similar and one 'different' triangle. Which triangle is 'different'?

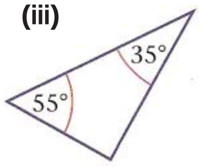
**a) (i)**



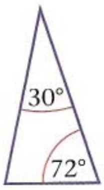

**(ii)**



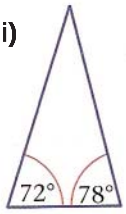

**(iii)**



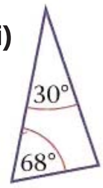

**b) (i)**



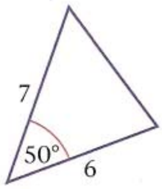

**(ii)**



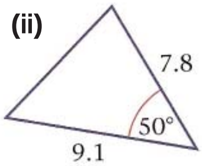

**(iii)**



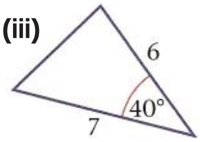

**c) (i)**




**(ii)**

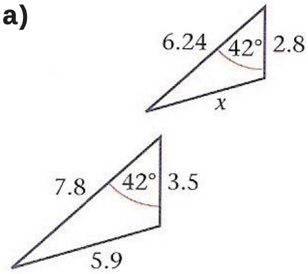



**(iii)**

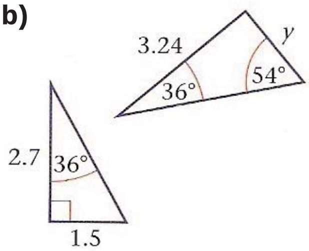



**v**. Write down why each of these pairs of triangles are similar. Calculate the length of each side marked by a letter.

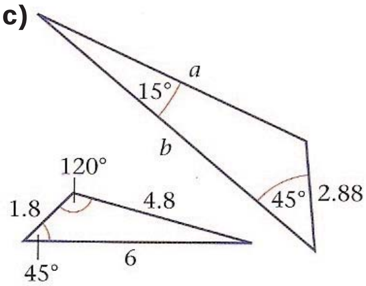
**a)**




**b)**




**c)**



# 2. Constructions

## 2.1 Constructing a triangle

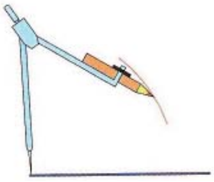
A **construction** is an accurate drawing carried out using a **straight edge** (**ruler**), a **pencil** and a **pair of compasses**.

**Example -** Construct a triangle with sides of length 3cm, 4cm and 6cm.

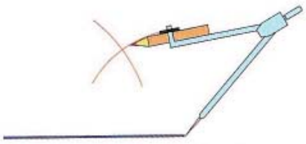
Follow the steps:

Draw the longest side using a ruler

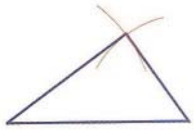

Set the compasses to 4cm. Draw an **arc**



Set the compasses to 3cm. Draw another arc

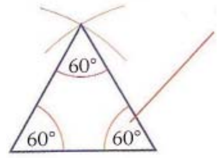


Join the ends of the lines where the arcs cross



**Example -** Use triangle constructions to draw an angle of 60$^{o}$ without using a **protractor**.

An equilateral triangle has angles 60$^{o}$. Construct an equilateral triangle by keeping the compasses set to the same length throughout.



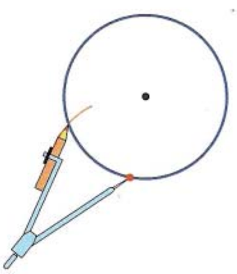

## 2.2 Constructing a regular hexagon

**Example -** Construct a regular hexagon.

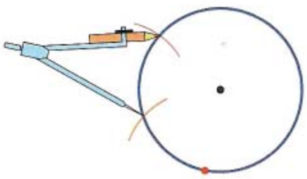
Keep your compasses set to the same length throughout.
Follow the steps:

Draw a circle of any **radius**:

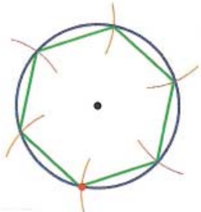
Put the pount of the compasses on the **circumference** and draw an arc



Place your compass point where the first arc crossed the circle

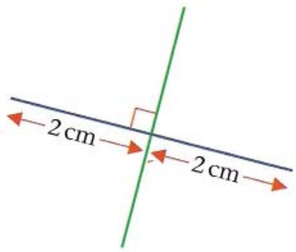


Draw four more arcs and join them to make a hexagon



## 2.3 Constructing perpendiculars

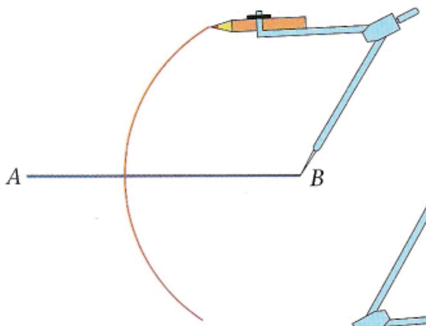
Perpendicular lines meet at right angles.



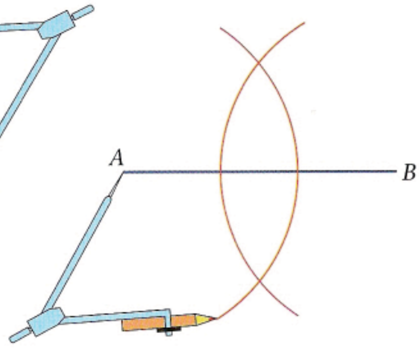

**Example -** Draw a **line segment** *AB* of length 8cm and construct its **perpendicular bisector**.

You need to set the compasses at more than half the length of the line segment, say 6cm.

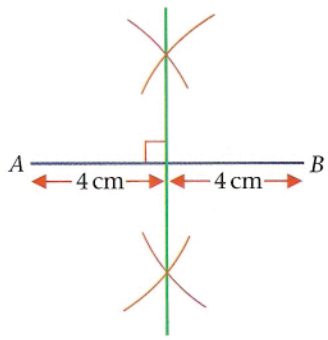
With the compass point at B, draw a large arc



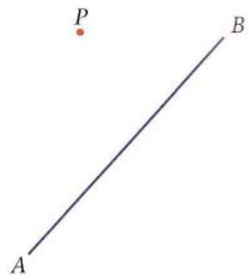

Place the compass point at A, draw a large arc




Join the points where the arcs cross. This is the perpendicular bisector of AB

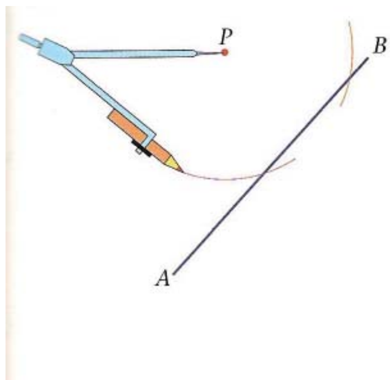



**Example -** Draw a line segment AB of length 8cm and construct its perpendicular bisector.

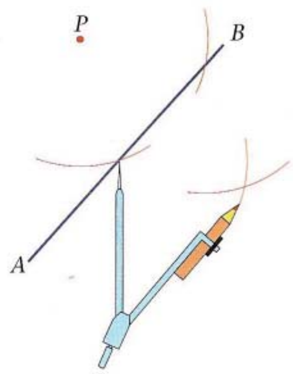



You need to set the compasses at more than half the length of the line segment, say 6cm.

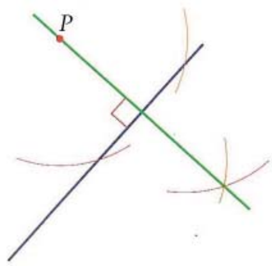
Place your compass point at P. Draw two arcs to the cut the line *AB:*



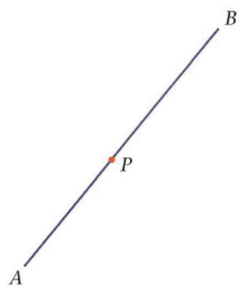

With the compass points at the points where the arcs cut the line, draw these arcs:




Join the points where the arcs cross to *P*. This is the line is perpendicular from *P* to *AB*.



**Example -** Draw a line segment AB of length 8cm and construct its perpendicular bisector.

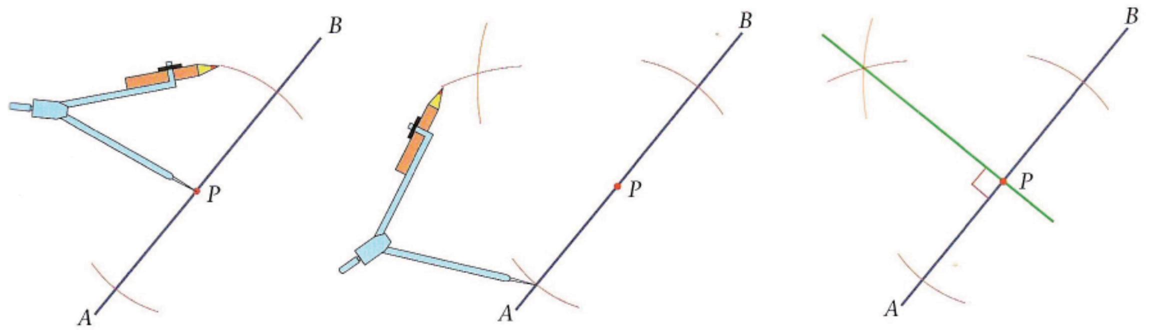



You need to set the compasses at more than half the length of the line segment, say 6cm.

Place your compass point at P. Draw two arcs to the cut the line *AB:*

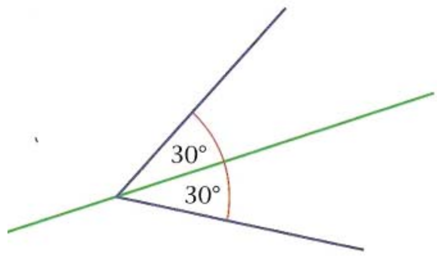
Open the compasses more. With the compass point at the points where the arcs cut the line, draw these arcs:

Join the point where the arcs cross to *P.* This line is perpendicular to *AB* at *P*.




## 2.3 Bisecting an angle

The **bisector** of an angle is the line which divides the angle into two equal parts.

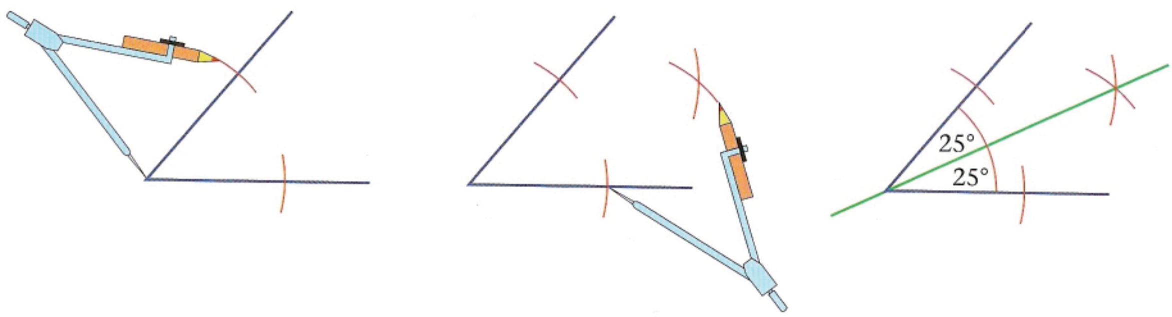



**Example -** Draw a 50° angle and construct its bisector.
Keep your compasses set to the same distance throughout.

Use a protractor to draw an angle of 50 . With your compass point at the **vertex** of the angle, draw two arcs to cut the sides of the angle:

Place your compass point at the points where the arcs cut the sides of the angle. Draw these arcs:

Join the point where the arcs cross to the vertex of the angle. This line is the bisector of the angle.



## Practice

**i.** Using a ruler, a pair of compasses and pencil only, construct triangles with sides of length

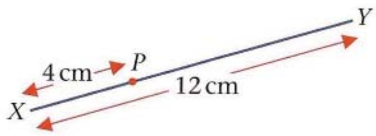
**a)** 4cm, 10cm and 9cm **b)** 8cm, 7cm and 12cm

**ii.** Use a ruler and protractor to draw triangles with the following lengths and angles

**a)** 6cm, 50° and 7cm **b)** 80°, 5.5cm and 58° **c)** 6cm, 5cm and 40°

**iii.** Draw a line segment of length 10cm. Using a straight edge, a pair of compasses and pencil only, construct the perpendicular bisector of this line segment.

**iv.** Draw this line accurately. Construct the perpendicular to *XY* at *P*.



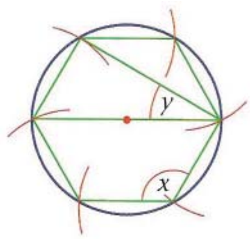

**v.** Draw a line segment AB and a point below it, Q. Construct the perpendicular from *Q* to *AB*.

**vi.** Construct an equilateral triangle with sides of length 8cm. What is the size of each angle of your construction?

**vii.** Draw an angle of any size. Without using any kind of angle measurer, construct the bisector of the angle.

**viii.** The diagram shows a construction of a regular hexagon. What is the size of

**a)** angle *x* **b)** angle *y*?



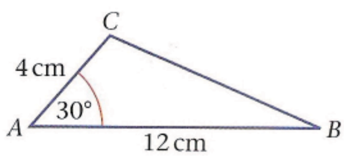

**ix.** Without using any form of angle measurer, construct an angle of

**a)** 90° **b)** 45° **c)** 135° **d)** 60° **e)** 30° **f)** 15° **g)** 120°

**x.** This diagram is a sketch of a triangle *ABC*.

**a)** Without using any form of angle measurer, construct the triangle *ABC*.

**b)** Measure the length of *BC*.



# 3. Measure and Mensuration

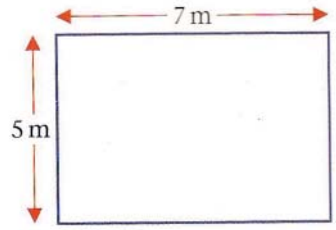
## 3.1 Introduction

**Example -** The dimensions of a rectangular lawn are given as:

**width** = 5 m, **length** = 7 m

Each measurement is given to the nearest metre.
**a)** Write the longest and shortest possible values for the width and the length of the rectangle.

**b)** Calculate the largest and smallest possible values for the **area** of the lawn.




**a)** The width is 5 m to the nearest metre. So the shortest value is 4.5 m and the longest is 5.5 m
The length is 7 m to the nearest metre. So the shortest value is 6.5 m and the longest is 7.5 m.

**b)** The area of a rectangle is width x length.

To find the smallest area we combine the shortest width and length. This gives

Area = 4.5 x 6.5 = 29.25 m$^2$.

To find the largest area we combine the longest width and length. This gives

Area = 6.5 x 7.5 = 41.25 m$^2$.

### Practice

**i.** A piece of carpet is rectangular. Its dimensions are quoted to the nearest 10cm as

width = 3.4 m, length = 4.6 m

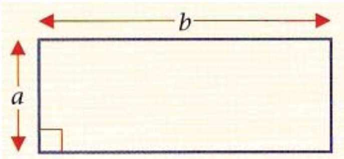
**a)** Write the shortest and longest widths for the carpet.
**b)** Write the shortest and longest lengths for the carpet.
**c)** Calculate the smallest and largest possible areas for the carpet.

## 3.2 Perimeter and area of triangles and quadrilaterals

You may have seen these formulae for calculating area befre

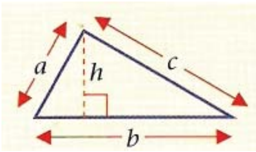
**Perimeter** = 2($a$ + $b$)

**Area** = $a$ x $b$ = $ab$



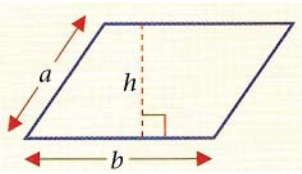

Perimeter = $a + b + c$

Area = 1/2 x **base** x **height** = 1/2 $bh$



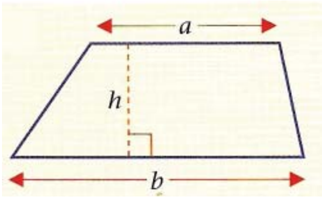

Perimeter = 2($a$ + $b$)

Area = $bh$

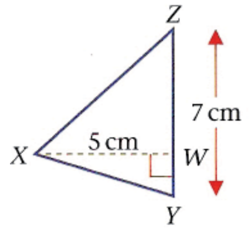



Perimeter = Sum of length of all four sides

Area = 1/2 ($a$ + $b$)$h$



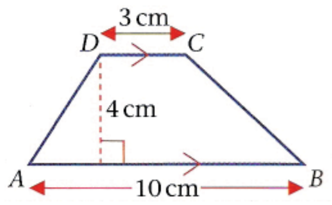
**Example -** Work out the area of triangle *XYZ*.




If you take *YZ* as the base, the height is *XW*.

The area of triangle *XYZ* is 1/2 x base x height = 1/2 x 7 x 5 = 17.5 $cm^2$

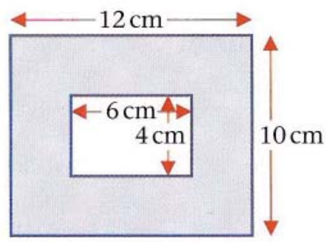
**Example -** Use the formula to calculate the area of the trapezium.




Area of trapezium = 1/2 $(a + b)h$ = 1/2(3 + 10) x 4

= 1/2 x 13 x 4 = 13 x 2 = 26 $cm^2$

**Example -** Find the shaded area.

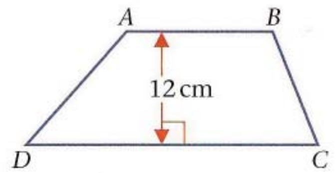



It is easiest to find the area of the largest rectangle and subtract the area of the smaller rectangle.

Large rectangle: 12 x 10 = 120 $cm^2$
Small rectangle: 6 x 4 = 24 $cm^2$

Area of shaded region: 120 - 24 = 96 $cm^2$

**Example -** *ABCD* is a trapezium with *AB* parallel to *DC*.

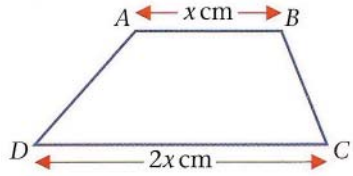



The lengths of *AB* and *CD* are in the ratio 1:2.
The perpendicualr distance between *AB* and *CD* = 12 cm.
The area of *ABCD* = 72 $cm^2$

Calculate the length *AB*.

If *AB* and *CD* are in the ratio 1:2 then *AB* = 1/2 *CD*.

If we let *AB* = $x$ cm then *CD* = $2x$ cm.



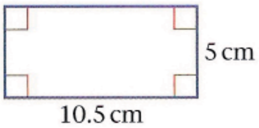

Area of *ABCD* = 1/2(*AB* + *CD*) x $h$ = 1/2($x + 2x$) x 12
= 1/2($3x$) x 12 = $18x$.

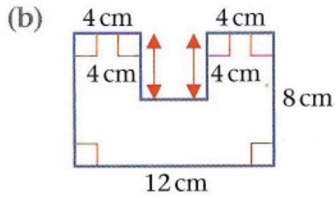
We know that Area *ABCD* = 72 $cm^2$. So, $18x = 72$. $x = 4$.

So, the length *AB* = 4 cm.

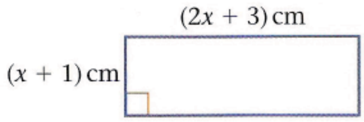
## Practice

**i.** Work out the area and perimeter of these shapes:




**ii.**
**a)** Find in its simplest form an expression for the perimeter of this rectangle in terms of x.
**b)** Given that the perimeter is 50 cm, calculate the value of x.
**c)** Calculate the area of the rectangle.

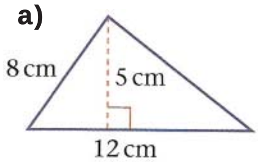



**iii.** The area of a square is numerically equal to the perimeter of the square in cm.
**a)** Calculate the length of a side of a square.
**b)** Calculate the area of the square.

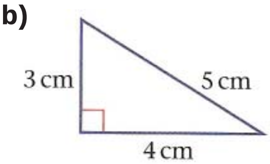
**iv.** A farmer uses exactly 1000 metres of fencing to fence off a square field. Calculate the area of the field.

**v.** Calculate the area of each triangle.

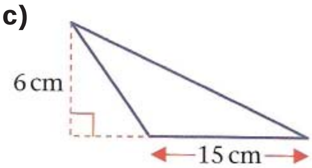
**a)**



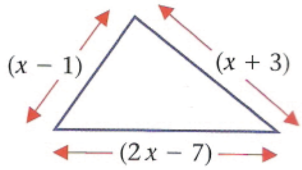

**b)**




**c)**

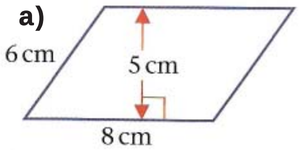



**vi.**
**a)** Write down an expression for the perimeter of a triangle in terms of x.
**b)** If the perimeter of the triangle is 29, calculate the value of x.

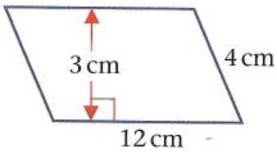



**vii.** Calculate the area of each parallelogram.

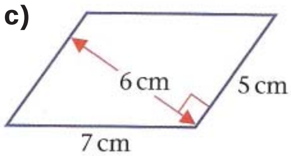
**a)**



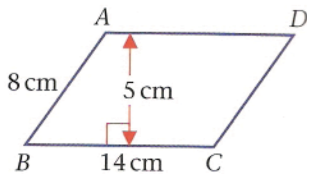

**b)**



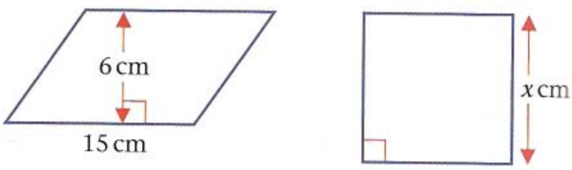

**c)**




**viii.** *ABCD* is a parallelogram. *BC* = *AD* = 14 cm, *AB* = *DC* = 8 cm.
**a)** Calculate the area of the parallelogram.
**b)** Calculate the perpendicular distance between *AB* and *DC*.

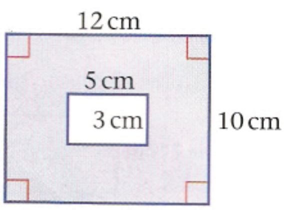



**ix.** The diagram shows a parallelogram and a square. These two shapes have equal areas. Calculate the value of *x* (the side of the square).

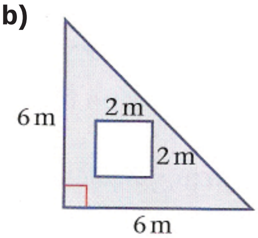



**x.** Find the shaded area in each diagram.

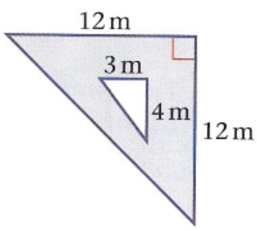
**a)**




**b)**




**c)**

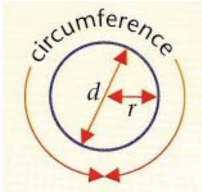


## 3.3 Circumference and area of circles

**Diameter** = 2 x radius or $d = 2r$

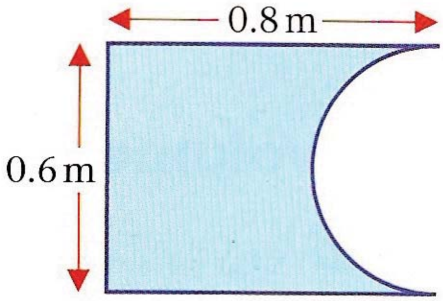
Circumference = $2\pi r = \pi d$

Area = $\pi r^2 = \pi d^2/4$




**Example -** The diagram shows one face of a sheet of metal in the form of a rectangle with a **semicircle** cut out from one end.

Calculate the area of the face of the sheet of metal.



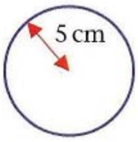

Radius of semicircle = 1/2 x 0.6 = 0.3 m

Shaded area = area of rectangle - area of semicircle
= $0.6 \times 0.8 - 1/2\,\pi\,(0.3)^2$
= $0.48 - 1/2\,\pi \times 0.09$
= $0.48 - 0.141$
= $0.339$ m$^2$ (to 3 d.p.)

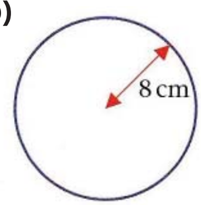
### Practice

**i.** Calculate the circumference and area of each circle.

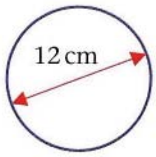
**a)**




**b)**




**c)**

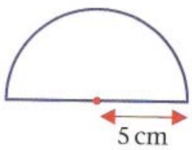



**ii.** The circumference of a circle in centimetres is numerically equal to the area of the circle in square centimetres. Show that the radius of the circle must be 2 cm.

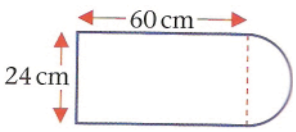
**iii.** The circumference of a circle is 15 cm. Find
**a)** The radius **b)** The area

**iv.** The area of a circle is 114 cm$^2$. Calculate the circumference of the circle.

**v.** Calculate the area and perimeter of the semicircle shown correct to 2 decimal places.




**vi.** The diagram represents a sheet of metal. It consists of a rectangle of length 60 cm and width 24 cm, and a semicircle. Calculate



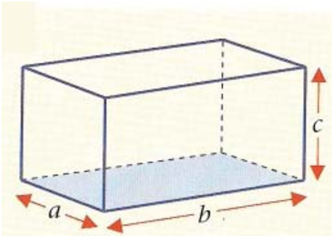

**a)** The perimeter of the sheet of metal.

**b)** The area of the sheet of metal.

## 3.4 Volume and area of 3-D shapes

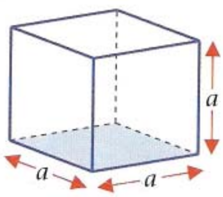
**Cuboid**

Total length around the edges = $4(a + b + c)$
**Surface area** = $2(ab + ac + bc)$
**Volume** = $abc$




**Cube**

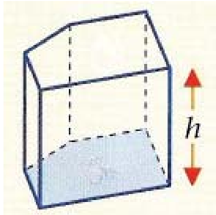
Total length around the edges = $12a$
Surface area = $6a^2$
Volume = $a^3$




**Prism**
For a general prism:

Surface area = 2 x area of base + total area of vertical faces

Volume = area of base x vertical height = area of base x $h$

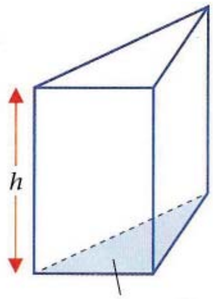



**Example -** Calculate the height of a prism which has a base area of 25 $cm^2$ and a volume of 205 $cm^3$

Volume = area of base x $h$

$205 = 25 \times h$

$h = 205/25 = 8.2$ cm




**Example -** *ABCDEF* is a triangle-based prism. The angle *ABC* = 90°.

$AB = x$ cm, $BC = (x + 3)$ cm, $CD = 8$ cm.
The volume of the prism = 40 $cm^3$.

Show that $x^2 + 3x - 10 = 0$.

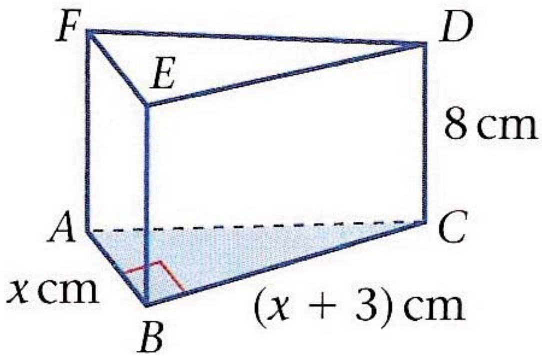
Volume of the prism = area of base x height

Area of base = 1/2 x $AB$ x $BC$ = 1/2 x $x$ $(x + 3)$

So the Volume = $1/2\, x(x + 3) \times 8 = 4x(x + 3)$.

We know the volume is 40 so, $4x(x + 3) = 40$
$x(x + 3) = 10$
$x^2 + 3x = 10$

Subtracting 10 from both sides, gives: $x^2 + 3x - 10 = 0$.



## Practice

**i.** Calculate the volume of a cube of side length

**a)** 5 m **b)** 12 cm **c)** 3.8 cm

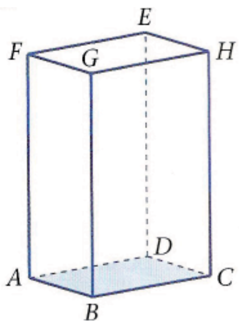
**ii.** A cube has a volume of 1000 cm$^3$. Calculate

**a)** the length of a side of a the cube **b)** the surface area of the cube

**iii.** Calculate the volume of a cuboid with sides of length

**a)** 5 cm, 6 cmand 6 cm **b)** 4.5 cm, 9.2 cm and 11.6 cm

**iv.** The volume of the cuboid *ABCDEFGH* is 384 cm$^3$. The edges *AB, BC* and *AF* are in the ratio: *AB : BC : AF* = 1 : 2 : 3

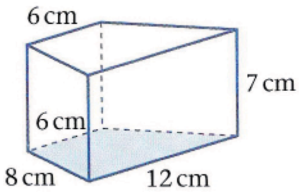


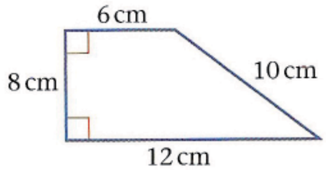

**a)** The length of *AB* **b)** The surface area of the cuboid

**v.** The diagram shows a prism and its base, which is a trapezium

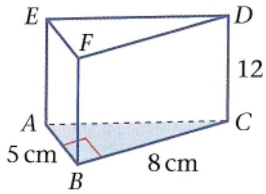
**a)** Calculate the volume of the prism

**b)** Calculate the surface area of the prism

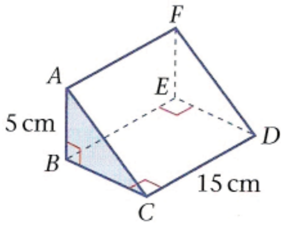


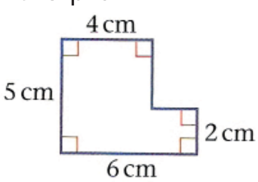

**vi.** *ABCDEF* is a triangle-based prism. Angle *ABC* = 90$^o$, *AB* = 5 cm, *BC* = 8 cm and *CD* = 12 cm. Calculate the volume of *ABCDEF*.

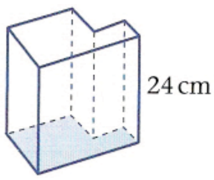



**vii.** *ABCDEF* is a wedge of volume 450 cm$^3$. Angle *ABC* = 90$^o$, *AB* = 5 cm, and *CD* = 15 cm. Calculate the length of *BC*.



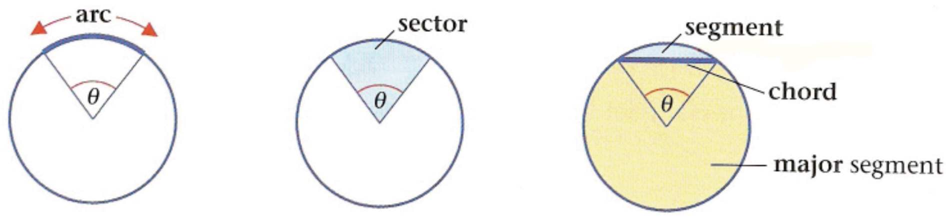

**viii.** This is the base of a prism of vertical height 24 cm. Calculate the volume of the prism.

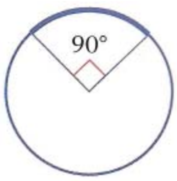


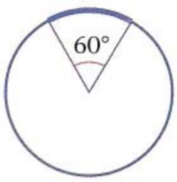
## 3.5 Finding the length of an arc of a circle

In the following three sections we will learn how to find the length of an arc of a circle, the area of a **sector of a circle** and the area of a **segment of a circle**.

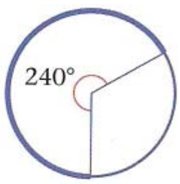



In this diagram the arc length is 1/4 of the circumference because the angle is 90°, which is 90/360 = 1/4 of a whole turn.



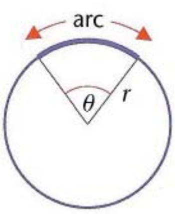

In this diagram the arc length is 1/6 of the circumference because the angle is 60°, which is 60/360 = 1/6 of a whole turn.




In this diagram the arc length is 2/3 of the circumference because the angle is 240°, which is 240/360 = 2/3 of a whole turn.

In general, if the angle at the centre is $\theta$ then

arc length = $\dfrac{\theta}{360}$ of the circumference = $\dfrac{\theta}{360} \times 2\pi r$

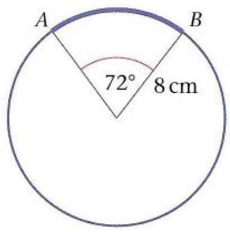



This gives the formula:

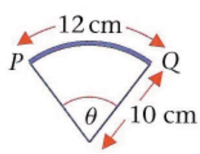
$$\text{Arc length} = \frac{2\pi r\theta}{360} = \frac{\pi r\theta}{180}$$

**Example -** Calculate the length of the arc *AB*.

**Arc length** = $\dfrac{\pi \times 8 \times 72}{180}$ = 10.05 (correct to 2 d.p.)




**Example -** The length of the arc *PQ* is 12 cm. Calculate the angle $\theta$



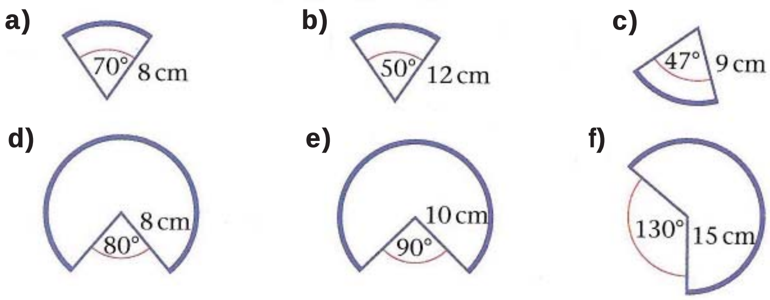

$$\text{Arc length} = \frac{\pi r\theta}{180}$$

so, $\dfrac{\pi \times 10 \times \theta}{180} = \dfrac{\pi \times \theta}{18} = 12$ . Multiply each side by 18 and divide each side by π:

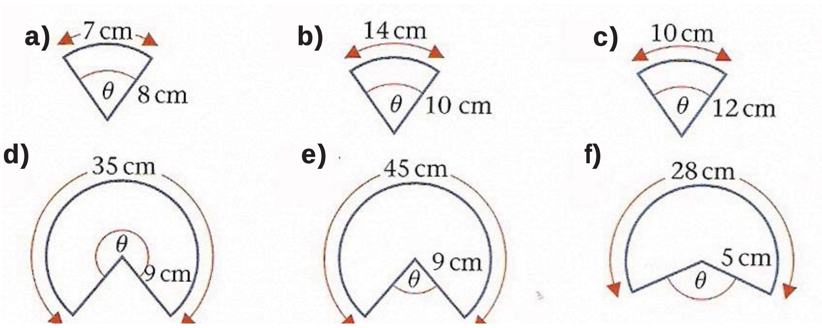
$$\text{So, } \theta = \frac{18 \times 12}{\pi} = 68.75^o$$

## Practice

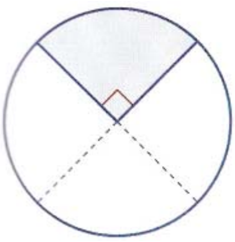
**i.** Calculate each of these arc lengths.



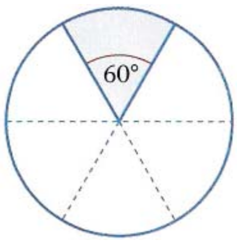

**ii.** Calculate each of the angles marked $\theta$.




## 3.6 Finding the area of a sector of a circle



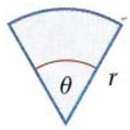
The shaded area is one quarter of the area of the circle because the angle is 90°, which is one quarter of a whole turn of 360°.




The shaded area is one sixth of the area of the circle because the angle is 60°, which is one sixth of a whole turn of 360°.

In general, if the angle at the centre is $\theta$ then

$$\text{arc length} = \frac{\theta}{360} \text{ of the circumference} = \frac{\theta}{360} \times \pi r^2$$

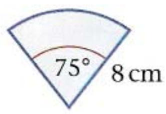



This gives the formula:

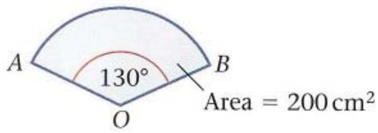
$$\text{Area of a sector} = \frac{\pi r^2 \theta}{360}$$

**Example -** Calculate the area of the sector:

$$\text{Area} = \frac{\pi r^2 \theta}{360}, \ r = 8. \text{ So, area} = \frac{\pi \times 64 \times 75}{360} = 41.89 \text{ cm}^2$$



**Example -** AOB is a sector of a circle. Angle AOB = 130°.
Area of the sector AOB = 200 cm$^2$.




Calculate the radius OA of the circle of which AOB is a sector.

$$\text{Area of a sector} = \frac{\pi r^2 \theta}{360} = 200$$

Multiply each side by 360 and divide each side by $\pi\theta$:

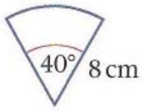
$$r^2 = \frac{200 \times 360}{\pi \times \theta} = \frac{200 \times 360}{\pi \times 130} = 176.2947$$

So, r = $\sqrt{176.2947} = 13.28\,\text{cm}$

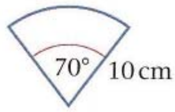
## Practice

**i.** Calculate the area of each of these sectors of circles:

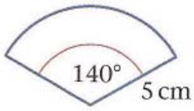
**a)**



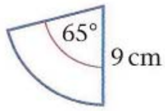

**b)**



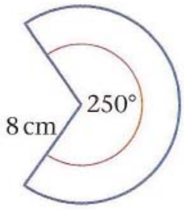

**c)**



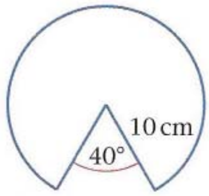

**d)**



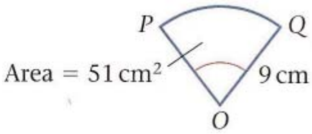

**e)**



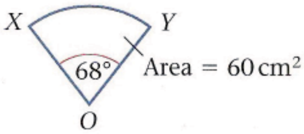

**f)**



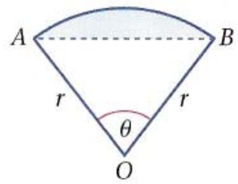

**ii.** *OPQ* is a sector of a circle centre *O* of radius 9 cm. The area of the sector *OPQ* is 51 cm$^2$. Calculate the size of the angle *POQ*.




**iii.** *OXY* is a sector of a circle centre *O*. The area of the sector *OXY* is 60 cm$^2$. The angle *XOY* = 68°. Calculate the length of the radius z of the circle.



## 3.6 Finding the area of a segment of a circle



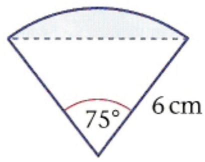

The area of the sector $OAB = \dfrac{\pi r^2 \theta}{360}$

For triangle $OAB$: Area of triangle $OAB$ = (1/2)$r$ x $r$ sin $\theta$.

The area of the shaded segment is the difference between these two areas:

$$\text{Area of segment} = \frac{\pi r^2 \theta}{360} - (1/2) r^2 \sin \theta.$$

**Example -** Calculate the area of the shaded segment of the circle.



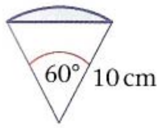

$$\text{Area of segment} = \frac{\pi r^2 \theta}{360} - (1/2) r^2 \sin \theta = \frac{\pi \times 6^2 \times 75}{360} - \frac{1}{2} \times 6^2 \times \sin 75^o$$

$$= 23.562 - 17.387 = 6.18\text{cm}^2$$

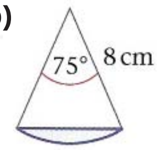
### Practice

**i.** Calculate the area of each shaded segment:

**a)**

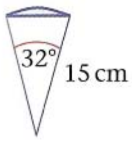



**b)**

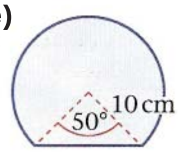



**c)**


**d)**



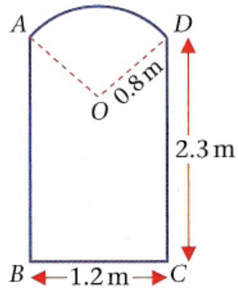

**e)**




**f)**


**ii.** A door is in the shape of a rectangle *ABCD* with a sector *OAD* of a circle.

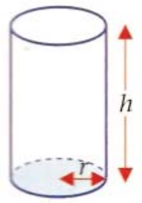
DC = AB = 2.3 m, BC = AD = 1.2 m and the radius of the circle is *OA* where OA = OD = 0.8 m. Calculate:




**a)** The perimeter of the door

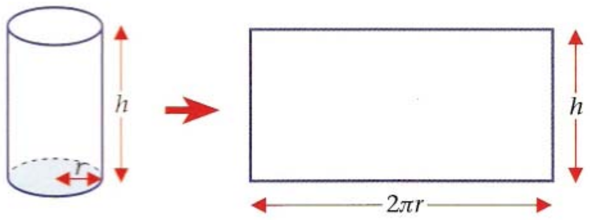
**b)** The area of the door

## 3.7 Finding volumes and surface areas




This cylinder has a circular base of radius r cm and a height of h cm.
Its surface area is made up of the area of the curved surface plus the areas of the circular top and base.

The area of the top and base is equal to $2\pi r^2$.
To find the area of the curved surface we 'unwrap' the cylinder and find the area of the rectangle:




The area of this rectangle is $2\pi rh$.

So the total surface area of the cylinder in cm$^2$ is

Surface area = $2\pi rh + 2\pi r^2$.

The volume of a cylinder is found using:

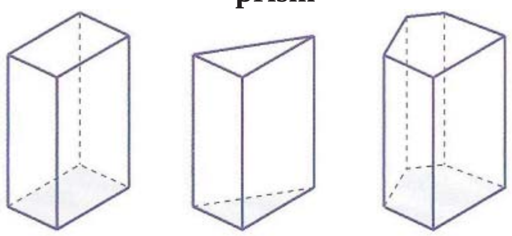
Volume = $\pi r^2 h$

**Example -** Calculate the surface area and volume of a cylinder with circular base of radius 12 cm and height 30 cm.

Surface area $= 2\pi rh + 2\pi r^2$
$= 2 \times \pi \times 12 \times 30 + 2 \times \pi \times 12^2$
$= 2261.947 + 904.778$
$= 3166.725 = 3167$ cm$^2$ (to 4 s.f.)

Volume $= \pi r^2 h$
$= \pi \times 12^2 \times 30$
$= 13{,}572$ cm$^3$ (to 5 s.f.)

A prism is a 3-D shape which has the same cross-section throughout its height.
Here are some examples:



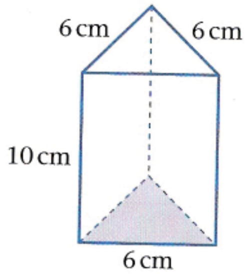

The volume of any prism is:
area of base x vertical height
or
area of **cross-section** x vertical height

The surface area of any prism is:
2 x area of base + total area of vertical faces

### Practice

**i.** Find the volume and surface area of a cylinder of height 4 cm and circular base of radius 5 cm.

**ii.** A prism of height 10 cm has a cross-sectional shape of an equilateral triangle of side 6 cm. Find the volume and surface area of the prism.

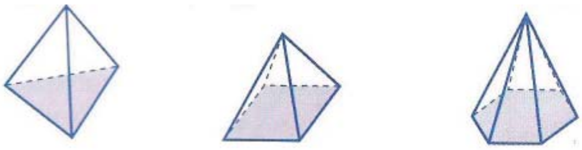



**iii.** A bin is the shape of a cylinder. Its base has radius 0.5 m and its height is 1.2 m. Find its volume.

## 3.8 Volume of a pyramid

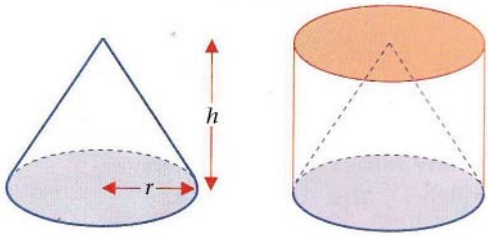
The shapes shown are all pyramids.
The base of a pyramid is a polygon.
The other edges are straight lines which lead to a point, called a vertex. The volume of a pyramid is given by:

volume = 1/3 x area of base x height




A cone is a pyramid with a circular base.

volume of a cone = 1/3 x area of base x height
= 1/3 $\pi r^2 h$

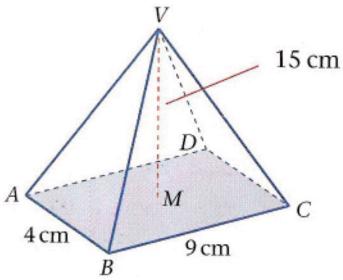



**Example -** A pyramid *VABCD* has a rectangular base *ABCD*. The vertext *V* is 15 cm vertically above the mid-point *M* of the base. *AB* = 4 cm and *BC* = 9 cm.

Calculate the volume of the pyramid.

The area of the base is 4 x 9 = 36 $cm^2$

so, volume of pyramid = 1/3 x area of base x vertical height
= 1/3 x 36 x 15 = 180 $cm^3$

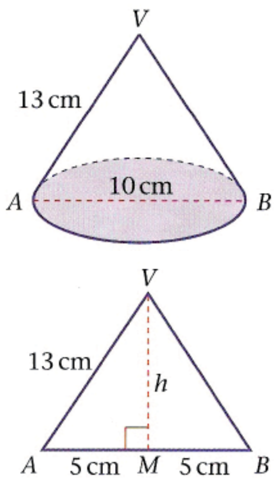



**Example -** The cone has a circular base of diameter *AB* length 10 cm. The slant height is 13 cm. Calculate the volume of the cone.

First, we need the vertical height from the mid-point of *AB* to *V*.
We can use Pythagoras' theorem:

$$AV^2 = h^2 + MA^2$$
$$h^2 = AV^2 - MA^2$$
$$= 13^2 - 5^2$$
$$= 169 - 25 = 144, \text{ so } h = 12$$

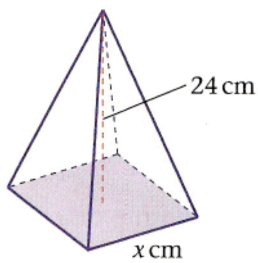
So, volume of cone = (1/3) $\pi r^2 h$
= 1/3 x π x $5^2$ x 12
= 1/3 x π x 25 x 12
= 314.2 $cm^3$ (correct to 1 d.p.)




**Example -** A pyramid has a square base of side *x* cm and a vertical height 24 cm. The volume of the pyramid is 392 $cm^3$. Calculate the value of *x*.

volume = 1/3 x area of base x height
= 1/3 x $x^2$ x $h$
392 = 1/3 x $x^2$ x 24

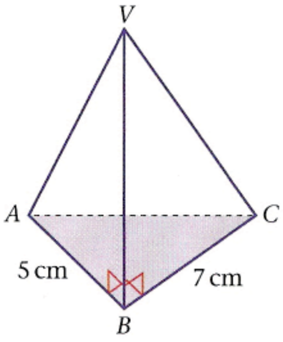
$x^2$ = (3 x 392)/24 = 49, $x$ = 7



**Practice**

**i.** *VABCD* is a square-based pyramid. The vertex V is 20 cm vertically above the mid-point of the horizontal square base *ABCD,* and *AB* = 12 cm.

**ii.** *VABC* is a triangle-based pyramid. The vertex *V* is vertically above the point *B*. The base, *ABC*, is a triangle with a right angle at *B*. *AB* = 5 cm, *BC* = 7 cm and *VC* = 25 cm. Calculate the volume of the pyramid *VABC*.



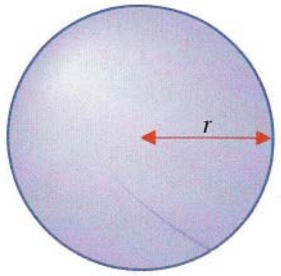

**iii.** A cone has a circular base of radius *r* cm. The vertical height of the cone is 15 cm. The volume of the cone is 600 $cm^3$. Calculate the value of *r*.

## 3.9 Surface area and volume of a sphere

For a **sphere** of radius $r$:

$$\text{volume of a sphere} = \frac{4\pi r^3}{3}$$

$$\text{surface area} = 4\pi r^2$$




**Example -** Calculate the volume of a sphere
of radius 5 cm.

$$\text{Volume of a sphere} = \frac{4 \times \pi \times 5^3}{3}$$

$$= \frac{4 \times \pi \times 125}{3} = 523.6 \text{ cm}^3 \text{ (correct to 1 d.p.)}$$

**Example -** The surface area of a sphere is 2000 $cm^2$. Calculate the radius of the sphere.

Using surface area = $4\pi r^2$, we have:

$$4\pi r^2 = 2000$$

$$r^2 = \frac{2000}{4\pi} = \frac{500}{\pi} = 159.15$$

So, r = 12.62 cm (correct to 2 d.p.)

**Practice**

**i.** Calculate the volume and surface area of a sphere

**a)** of radius 8 cm **b)** of radius 7.2 cm **c)** of diameter 19 cm **d)** of diameter x cm

**ii.** A sphere has volume of 5000 $cm^3$. Calculate the radius of the sphere.

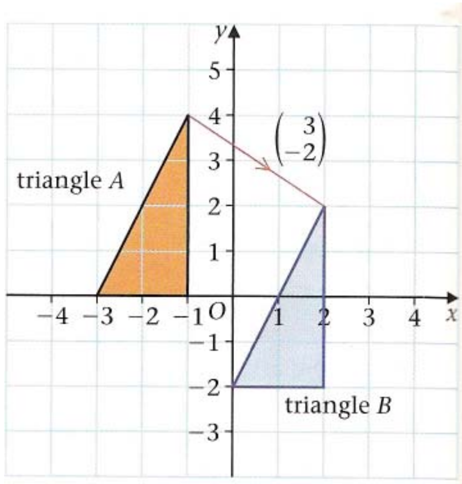
**iii.** A cube of side x cm and a sphere of radius 6 cm have equal volumes. Calculate the value of x.

# 4. Transformations

A **transformation** is a change in an object's position or size. In this chapter we will learn three kinds of transformation - **translation**, **reflection**, **rotation**.

## 4.1 Translation

A translation moves every point on a shape the same distance and direction.




In the diagram triangle $A$ is translated to $B$. $B$ is the **image** of $A$. All the points on A are moved +3 units parallel to the $x$-axis followed by -2 units parallel to the $t$-axis.

The translation is described by the **vector** $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

In the vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

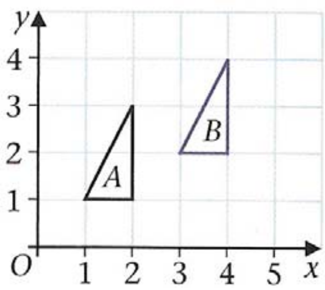
- $x$ gives the movement parallel to the ***x*-axis**.
- $y$ gives the movement parallel to the ***y*-axis**.

To describe a translation fully you need to give the distance moved and the direction of the movement. You can do this by giving the vector of the translation.

**Example -** Triangle $B$ is a translation of triangle $A$.
Describe the translation that takes A to $B$.

Triangle A has moved 2 squares in the $x$-direction
1 square in the $y$-direction.

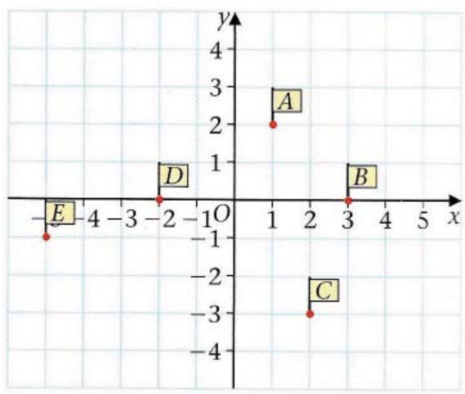
The translation has a vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.



### Practice

**i.** Write down the vectors describing these translations:

**a)** flag A to flag B
**b)** flag B to flag A
**c)** flag D to flag B
**d)** flag C to flag E
**e)** flag A to flag E.



**ii.** Use graph paper or squared paper. Draw a set of axes and label each one from - 5 to 5. Use the same scale for each axis. Draw a trapezium $A$, with vertices at (-5, 3), (-4, 3), (-3, 2), (-3, 1).

**a)** Transform $A$ by the translation $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$. Label the new trapezium $B$.

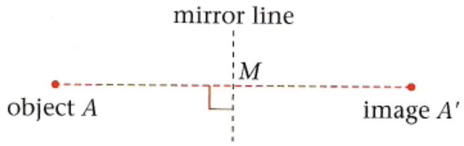
**b)** Transform $B$ by the translation $\begin{pmatrix} -1 \\ -6 \end{pmatrix}$. Label the new trapezium $C$.

**c)** Write the single translation vector required to transform $A$ to $C$.

## 4.2 Reflection

A **reflection** is a line which produces a mirror image.

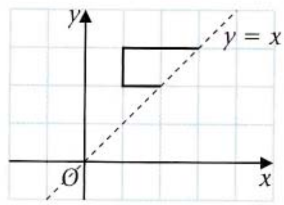
The mirror line is a line of **symmetry**.
The diagram on the right shows a point $A$ reflected in a line.



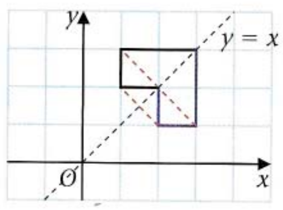

The original shape $A$ and the image $A'$ are the same distance from the line, but on opposite sides.

**Example -** Copy the diagram on squared paper.

**a)** Draw the reflection of the shape in the line $y = x$.
**b)** Reflect the image back in the line $y = x$. What do you notice?




**a)** Take each vertex on the object and locate its image. Imagine a line perpendicular to the line of symmetry. Join the points to produce the image.



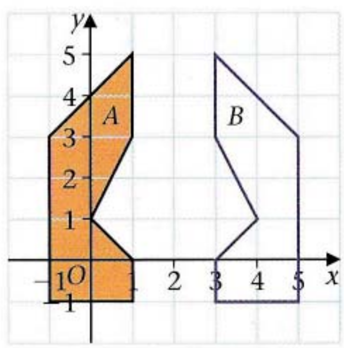

**b)** The image reflected in the line $y = x$ returns back to the object position.

### Practice

**i.** Draw a coordinate grid with both x- and $y$- axes going from -4 to 4.

**a)** Plot the points $P$(-2, 1), $Q$(-2, 1), $R$(-2, 1), $S$(-2, 1) and $T$(-2, 1) and join them in order to make a shape $PQRST$.
**b)** Draw the image of $PQRST$ after a reflection in the x-axis.

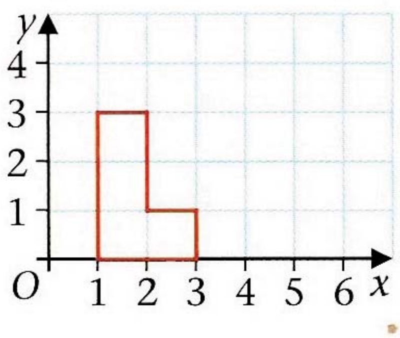
**ii.** Describe fully the transformation that maps shape $A$ onto $B$.




**iii.**
**a)** Copy this diagram. Extend the x- and $y$- axes to -5. Reflect the object in the line $x = 3$.

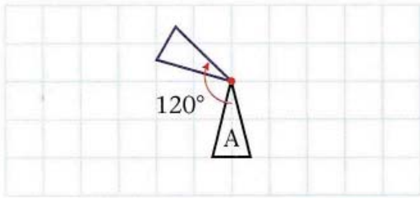
**b)** Reflect the image in the line $y = 2$.

**c)** Reflect the object in the line $x = 3$.



## 4.3 Rotation

An object can be turned around a point.
This point is called a **centre of rotation**.



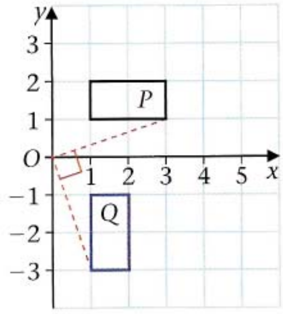

To describe a rotation fully you need to give the

- centre of rotation
- amount of turn
- direction of turn

**Example -** Describe fully the transformation which maps shape *P* onto shape *Q*.

Rotation of 90° **clockwise** about (0, 0).

Each vertex of the rectangle has been rotated 90° clockwise.




### Practice

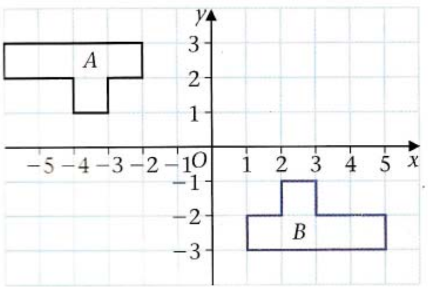
**i.** Use graph or squared paper. Draw a set of axes and label each from -5 to 5. Use the same scale for each axis. Draw a triangle with vertices (3, 1), (5, 1) and (5, 3). Label the triangle *A*.

**a)** Rotate A about the origin through 90° anticlockwise.

**b)** Rotate B about the origin 90° **anticlockwise**.

**c)** Write a single rotation to transform *A* to *C*.

**ii.** Describe the transformation which maps shape *A* onto shape *B*.



## 4.4 Combined transformations

We can combine transformations by performing one transformation and then performing another on the image.

**Example -**

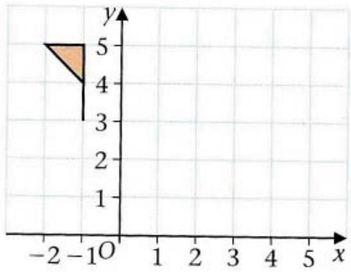
**a)** Reflect the flag in the $y$-axis

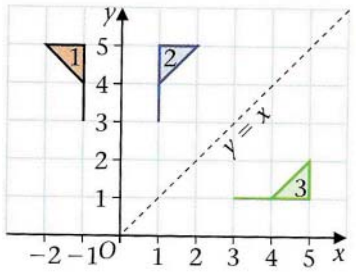
**b)** Reflect the image in the line $y = x$

**c)** Describe the single transformation to replace a) and b)

**d)** Reflect the image from a) in the line $x = 2$

**e)** Describe the single transformation to replace a) and d)

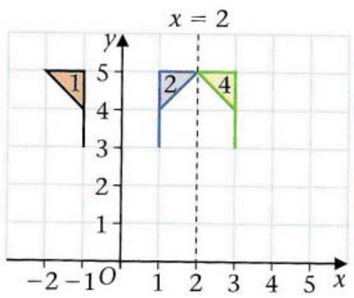





**a)** Flag 2 is the image of flag 1.

**b)** Flag 3 is the image of flag 2.

**c)** Single transformation:
rotate flag 1 90° clockwise about the origin.




**d)** The image from a) is flag 2. Flag 4 is the image of flag 2.

**e)** Single transformation: translate flag 1 by $\begin{pmatrix}4\\0\end{pmatrix}$.

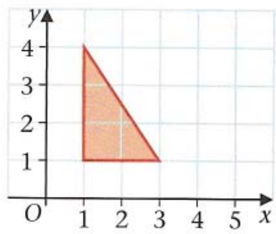
### Practice

**i.**

**a)** Reflect the triangle in the x-axis

**b)** Reflect the image in the $y$-axis

**c)** Describe the single transformation that replaces a) and b).

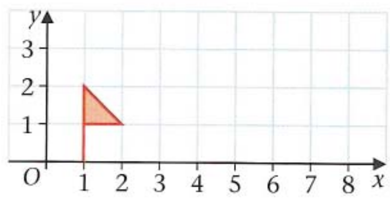


**ii.**

**a)** Reflect the shape in the line x = 3

**b)** Reflect the image in the line x = 6

**c)** Describe the single transformation that replaces a) and b).

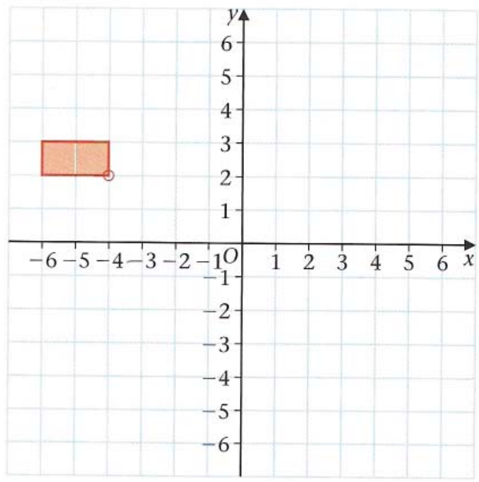


**iii.**

**a)** Reflect the rectangle in the line y = -x

**b)** Rotate the image 90° clockwise about the origin

**c)** Describe the single transformation that replaces a) and b).



**iv.** Use the same diagram at question iii.

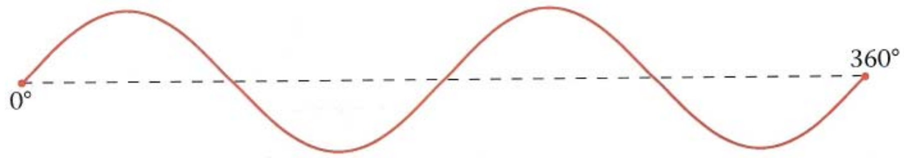
**a)** Rotate rectangle 90° clockwise about the origin

**b)** Reflect the image in the line y = -x

**c)** Describe the single transformation that replaces a) and b).

# 5. Loci

This path is called the **locus** of a point.

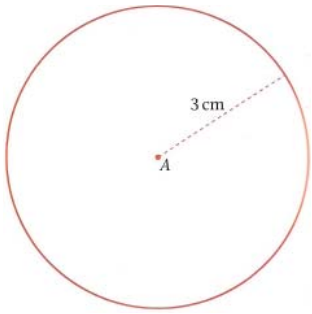



A locus is a set of points which obey a particular rule.

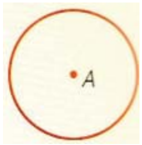
A locus may be produced by something moving according to a set of rules,or by a set of points which follow a mathematical rule.

**Example -** Draw the locus of a point which moves so that it is always 3 cm from a fixed point.

*A* is the fixed point. Draw a circle, radius 3 cm and centre at *A*. The locus of points satisfying the rule is the circumference of the circle.



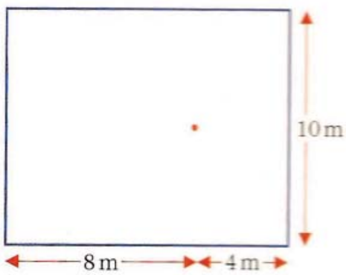

The locus of a point which moves so that it is always a fixed distance from a point *A* is a circle, centre *A*.



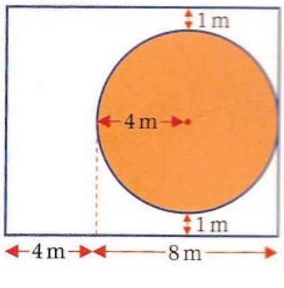

**Example -** The diagram shows a rectangular field.

A watering machine rotates from a fixed point on the field. Water from the machine reaches 4 m.

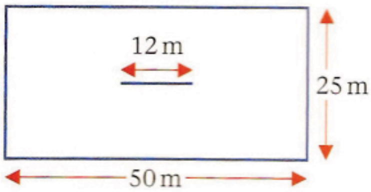
Complete the diagram and show the parts of the field watered by the machine.



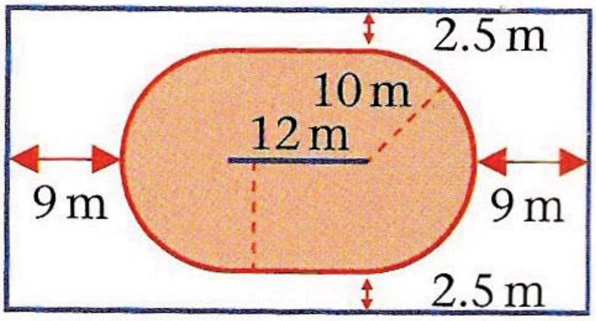

The machine waters the area within a circle, centre the machine, radius 4 m.



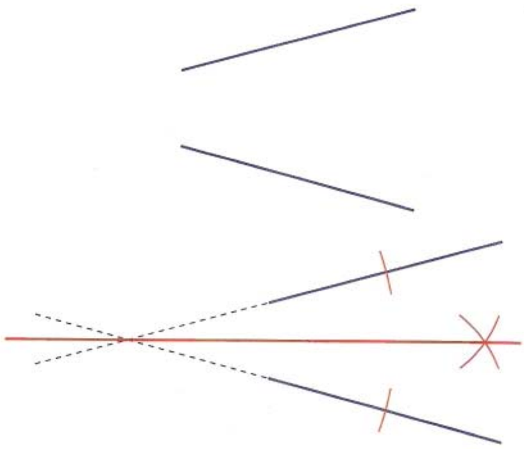
**Example -** A goat is tied in a field by a 10 m rope. The rope can slide along a bar 12 m long. Make a **scale drawing** to show the parts of the field the goat can graze.




The goat can graze in any of the shaded area




**Example -** Draw the locus of a point so that it is the same distance from two straight lines.

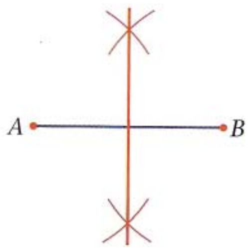


To do this we continue the two lines and then bisect the angle between them.

The perpendicular distance from this angle bisector to the two line is always the same.

**Example -** Draw the locus of a point that moves so that it is always the same distance from points *A* and *B*.

This locus is given by the perpendicular bisector of *AB*.




## Practice

**i.** Mark two points *A* and *B* roughly 4 cm apart. Draw a path equidistant from *A* and *B*.

**ii.** Draw the locus of a point that moves so that it is always 2.5 cm from a line 4 cm long.

**iii.** A running track is designed so that any point on the track is 22.3 m from a fixed line 150 m long.

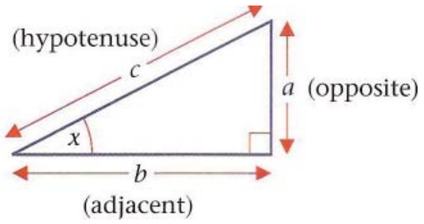
**a)** Draw the locus of the point.
**b)** Calculate the distance once round the running track.

# 6. Trigonometry

## 6.1 Trigonometric ratios

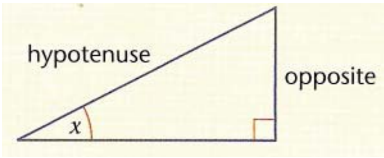
In any right angled triangle you can name the sides in relation to the angles:

- side $a$ is **opposite** to angle $x$
- side $b$ is **adjacent** to angle $x$
- side $c$ is the **hypotenuse** (opposite the right angle)

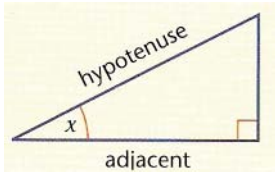



Using geometry we can prove that:

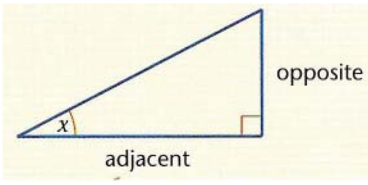
$$\sin x = \frac{\text{length of side opposite } x}{\text{length of hypotenuse}} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$



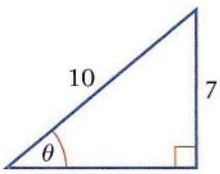

$$\cos x = \frac{\text{length of side adjacent to } x}{\text{length of hypotenuse}} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

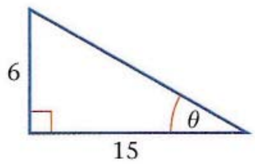


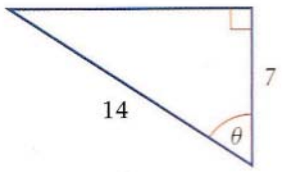

$$\tan x = \frac{\text{length of side opposite } x}{\text{length of side adjacent to } x} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{opp}}{\text{adj}}$$




**Example -** Write down which trigonometric ratio is needed to calculate angle $\theta$ in each of these triangles:




**a)** The given sides are opposite to angle $\theta$ and the hypotenuse so **sine** is needed.

**b)** The given sides are oppositee and adjacent to angle $\theta$ so **tangent** is needed.

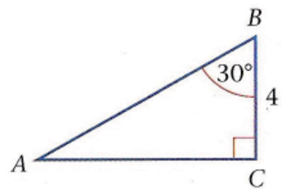
**c)** The given sides are adjacent to angle $\theta$ and hypotenuse so **cosine** is needed.

**Example -** Write down which trigonometric ratio is needed to calculate the side $AB$.

Side $BC$ is adjacent to the given angle. Side $AB$ is the hypotenuse.

So the ratio needed is cosine.

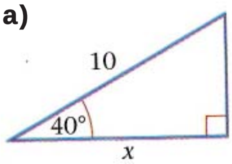
$$\cos x = \frac{\text{adj}}{\text{hyp}}$$



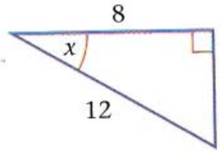
## Practice

**i.** Write down which trigonometric ratio is needed to calculate the side or angle marked x in each of these triangles.

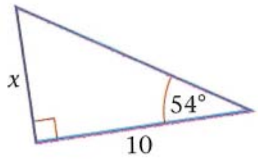
**a)**



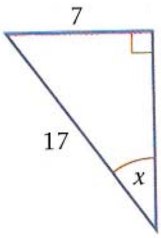

**b)**



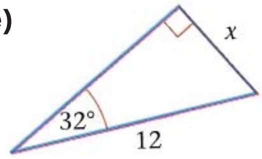

**c)**



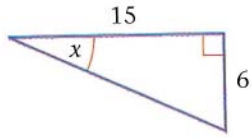

**d)**



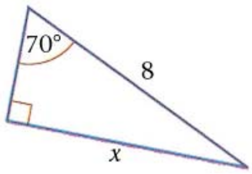

**e)**



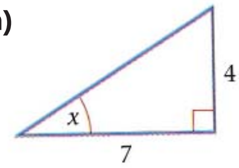

**f)**



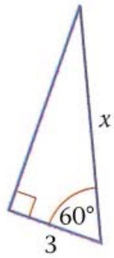

**g)**



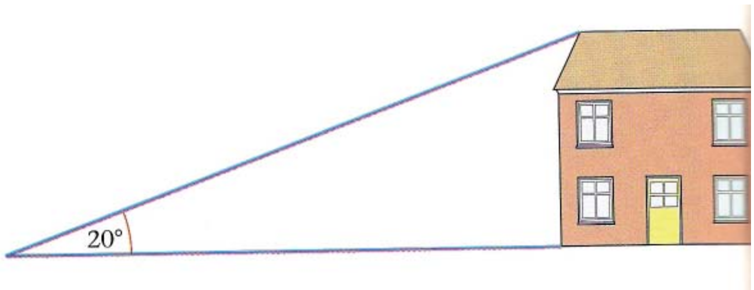

**h)**




**i)**




**ii.** Kler Paw stands 25 metres away from her new house in America which is built on flat ground. She uses a clinometer to measure the angle between the ground and the top of the house. The angle is $20^{o}$. Estimate the height of Kler Paw's house (use the fact that tan $20^{o}$ = 0.36 to 2 d.p.).

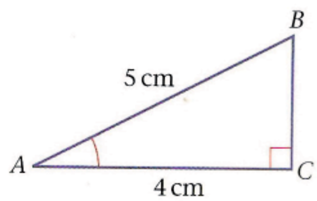



## 6.2 Using trigonometric ratios to find angles

**Example -** Calculate the size of the angle at A.

Since 4 is adjacent to A and 5 is the hypotenuse we use the cosine ratio:

$$\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5} = 0.8$$



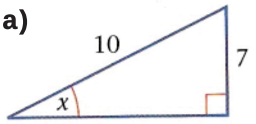

So, $A = \cos^{-1} 0.8 = 36.87^{o}$ (to 2 d.p.).

## Practice

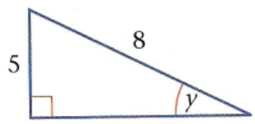
Calculate each of the angles marked with a letter by using the correct trigonometric ratio.

$\tan^{-1}(1/3) = 18.4^o$, $\cos^{-1}(2/3) = 48.2^o$, $\sin^{-1}(7/10) = 44.4^o$, $\tan^{-1}(2/3) = 21.8^o$, $\cos^{-1}(5/12) = 65.4^o$, $\sin^{-1}(5/8) = 38.7^o$, $\cos^{-1}(7.2/11.8) = 52.4^o$, $\sin^{-1}(1/2) = 30^o$, $\sin^{-1}(7/17) = 24.3^o$.

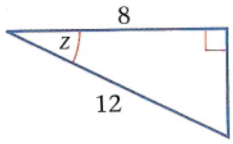
**a)**



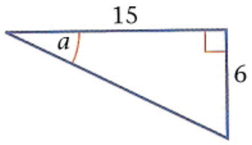

**b)**



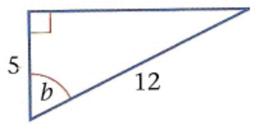

**c)**



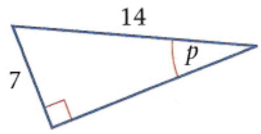

**d)**



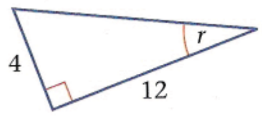

**e)**



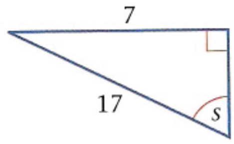

**f)**



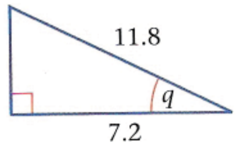

**g)**




**h)**




**i)**



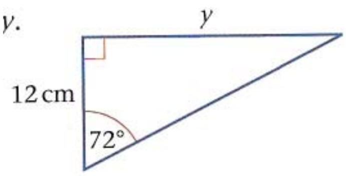

## 6.3 Using trigonometric ratios to find the length of sides

**Example -** Calculate the length of the side marked *y*.

For the given angle, *y* is opposite and 12 cm is adjacent.
So we use the tangent ratio:

$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{y}{12}$$

*y*.



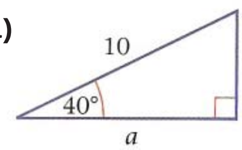

So, $y = 12 \times \tan 72^o = 12 \times 3.0777 = 36.93$ cm (to 2 d.p.)

## Practice

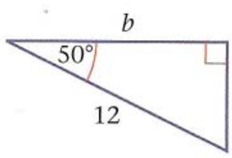
Calculate each length marked with a letter. Choose one of the trigonometric values from the box below to find each length.

cos (50) = 0.64, cos (40) = 0.77, sin (60) = 0.87, sin (70) = 0.94,
tan (24) = 0.45, tan (34) = 0.67, sin (32) = 0.53, cos (30) = 0.87.

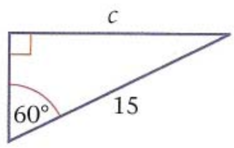
**a)**



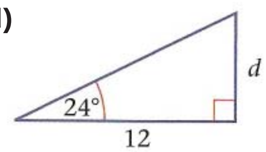

**b)**



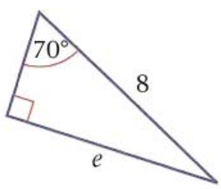

**c)**



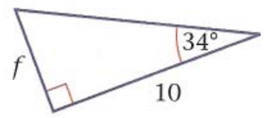

**d)**



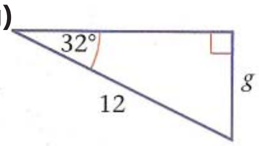

**e)**



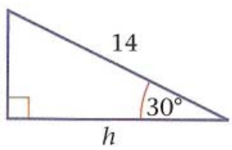

**f)**




**g)**




**h)**



# Glossary of Keywords

Here is a list of Mathematical words from this module. The section where the word appears is given in brackets. Find the words and what they mean - your teacher will test your memory soon!

| | |
|---|---|
| Right angle | (1.1) |
| Perpendicular | (1.1) |
| Acute angle | (1.1) |
| Obtuse angle | (1.1) |
| Reflex angle | (1.1) |
| Supplementary angles | (1.1) |
| Corresponding angles | (1.1) |
| Alternate angles | (1.1) |
| Co-interior angles | (1.1) |
| Triangle | (1.2) |
| Equilateral triangle | (1.2) |
| Isoceles triangle | (1.2) |
| Scalene triangle | (1.2) |
| Quadrilateral | (1.3) |
| Square | (1.3) |
| Parallel | (1.3) |
| Parallelogram | (1.3) |
| Trapezium | (1.3) |
| Kite | (1.3) |
| Diagonal | (1.3) |
| Rectangle | (1.3) |
| Rhombus | (1.3) |
| Arrowhead | (1.3) |
| Adjacent | (1.3) |
| Pentagon | (1.3) |
| Hexagon | (1.3) |
| Octagon | (1.4) |
| Polygon | (1.4) |
| Regular polygon | (1.4) |
| Exterior angle | (1.4) |
| Congruent | (1.5) |
| Hypotenuse | (1.5) |
| Vertices | (1.5) |
| Similar shapes | (1.6) |
| Enlargement | (1.6) |
| Scale factor | (1.6) |

| | |
|---|---|
| Construction | (2.1) |
| Straight edge | (2.1) |
| Ruler | (2.1) |
| Pencil | (2.1) |
| Pair of compasses | (2.1) |
| Arc | (2.1) |
| Protractor | (2.1) |
| Radius | (2.2) |
| Circumference | (2.2) |


| | |
|---|---|
| Construction | (2.1) |
| Straight edge | (2.1) |
| Ruler | (2.1) |
| Pencil | (2.1) |
| Pair of compasses | (2.1) |
| Arc | (2.1) |
| Protractor | (2.1) |


| | |
|---|---|
| Radius | (2.2) |
| Circumference | (2.2) |
| Line segment | (2.3) |
| Bisector | (2.3) |
| Vertex | (2.4) |

| | |
|---|---|
| Width | (3.1) |
| Area | (3.1) |
| Perimeter | (3.1) |
| Base | (3.2) |
| Height | (3.2) |
| Circumference | (3.2) |
| Diameter | (3.3) |
| Semicircle | (3.3) |
| Surface area | (3.4) |
| Volume | (3.4) |
| Sector of a circle | (3.5) |
| Segment of a circle | (3.5) |
| Arc length | (3.5) |
| Cuboid | (3.7) |
| Triangular prism | (3.7) |
| Pentagonal prism | (3.7) |
| Cross-section | (3.7) |
| Circumference | (3.7) |
| Triangle based pyramid | (3.8) |
| Square based pyramid | (3.8) |
| Hexagonal based pyramid | (3.8) |
| Sphere | (3.9) |

| | |
|---|---|
| Transformation | (4.1) |
| Translation | (4.1) |
| Rotation | (4.1) |
| Reflection | (4.1) |
| Image | (4.1) |
| Vector | (4.1) |
| x-axis | (4.1) |
| y-axis | (4.1) |
| Symmetry | (4.2) |
| Centre of rotation | (4.3) |
| Clockwise | (4.3) |
| Anticlockwise | (4.3) |
| Loci | (5) |
| Locus | (5) |

| | |
|---|---|
| Trigonometry | (6.1) |
| Sine | (6.1) |
| Cosine | (6.1) |
| Tangent | (6.1) |

# Assessment

This assessment is written to test your understanding of the module. Review the work you have done before taking the test. Good luck!

## Part 1 - Vocabulary

These questions test your knowledge of the keywords from this module. Complete the gaps in each sentence by using the words in the box.

| | | | | |
|---|---|---|---|---|
| **congruent** | **bisector** | **vectors** | **circumference** | **supplementary** |
| **rotation** | **scalene** | **enlargement** | **locus** | **perpendicular** |

**a)** We describe transformations to shapes using ___________________

**b)** Two lines which are at right angles to each other are ___________________

**c)** A triangle which has no sides or angles equal is a ___________________ triangle

**d)** Two shapes are ___________________ if they are exactly the same shape and size

**e)** The line which divides an angle exactly in two is the ___________________

**f)** Two shapes are similar if one is an ___________________ of the other

**g)** The distance around a circle is the ___________________

**h)** A ___________________ is a set of points which obey a rule

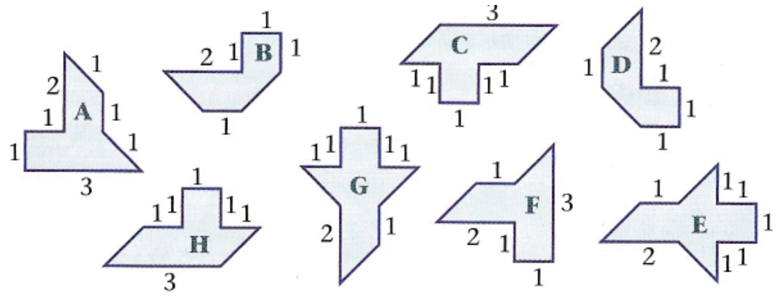
**i)** A ___________________ is when we turn an object around a point

**j)** If two angles sum to $180^o$ then they are ___________________

## Part 2 - Mathematics

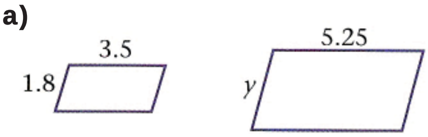
These questions test your understanding of the Mathematics in this module. Try to answer all the questions. Write your calculations and answers on separate paper. Where needed use $\pi = 3.14$.

**1.** Which pairs of shapes are congruent?

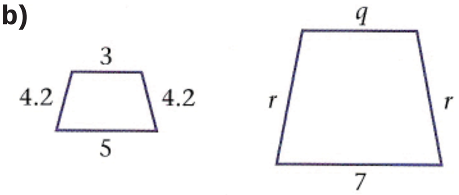


**2.** Each pair of shapes is similar. Calculate each length marked by a letter.

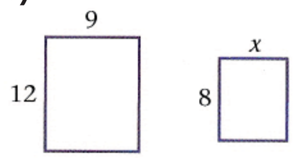
**a)**



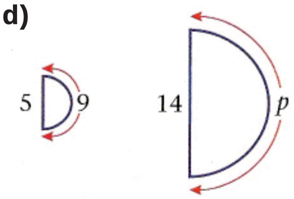

**b)**




**c)**



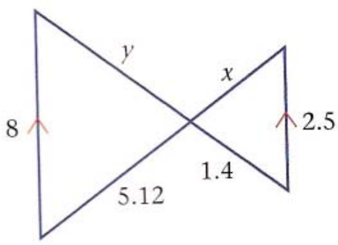

**d)**




**3.**

**a)** Explain why the two triangles in this diagram are similar.

**b)** Calculate the lengths of x and y.



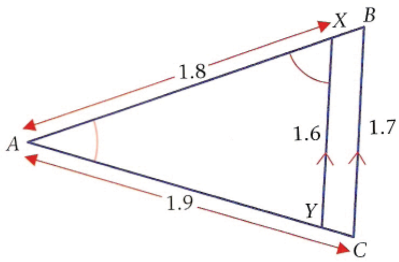

**4.**

**a)** Name the similar triangles

**b)** Explain why they are similar

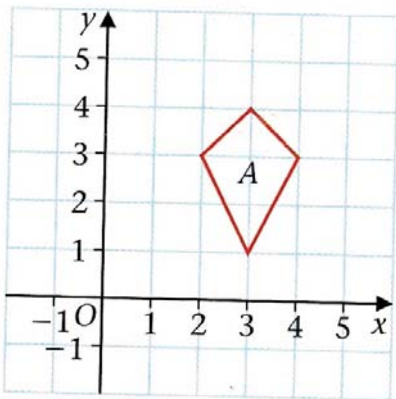
**c)** Calculate the length marked *AB*.

**d)** Calculate the length *AY*.

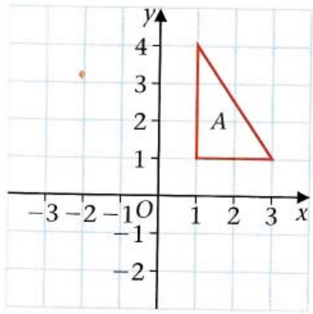



**5.** Draw the diagram below on squared paper.

Reflect shape *A* in the x-axis to give shape *B*.
Draw and label shape *B*.



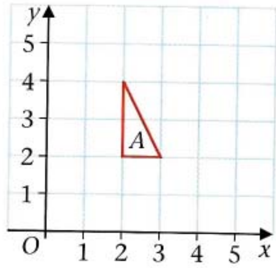
**6.** Make a copy on squared paper of the diagram.




Shape *A* is rotated 90° anticlockwise centre (0,1) to shape *B*.
Shape *B* is rotated 90° anticlockwise centre (0,1) to shape *C*.
Shape *C* is rotated 90° anticlockwise centre (0,1) to shape *D*.

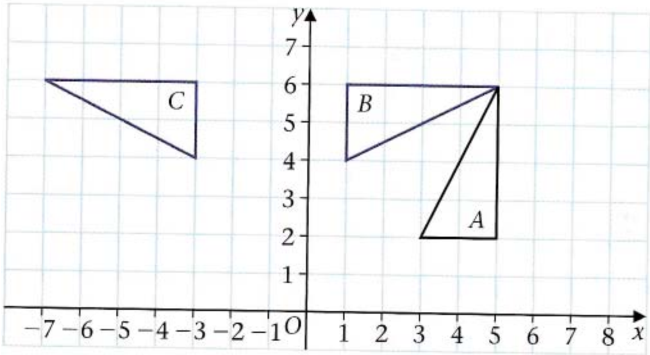
**a)** Mark the positions of shapes *B, C* and *D*.
**b)** Describe the single transformation that takes shape *C* to shape *A*.

**7.** Make a copy on squared paper of the diagram.




**a)** Rotate the triangle *A* 180° about *O*. Label your new triangle *B*.

**8.** Triangle *B* is a reflection of triangle *A*.

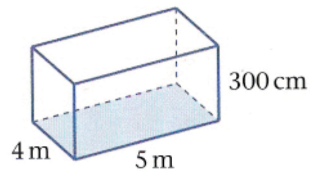



**a)** Copy the diagram on squared paper and draw the line of reflection.
**b)** Write down the equation of the line of reflection.
**c)** Describe fully the single transformation that maps triangle *A* onto triangle *C*.

**9.** Make a copy of the line *XY*. Draw the locus of all points which are 3 cm away from the line *XY*.

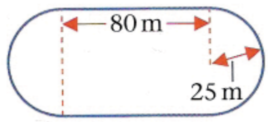

**10.** Construct a regular pentagon and a regular octogon of any size.

**11.** Here is a cuboid. The rectangular base has width 4 m and length 5 m. The height is 300 cm. The dimensions are all quoted to the nearest metre.



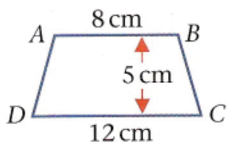

**a)** Calculate the maximum possible value of the cuboid.
**b)** Calculate the minimum possible value of the cuboid.

**12.** The diagram represents the plan of a sports field. The field is in the shape of a rectangle with semicircular pieces at each end.




Calculate:
**a)** The perimeter of the field.
**b)** The area of the field.

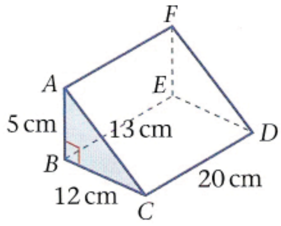
**13.** Calculate the area of the trapezium *ABCD*.




**14.** The circumference of a circle is 44 cm. Calculate the area of the circle.

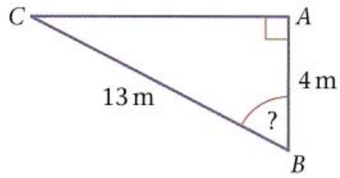
**15.** Calculate the volume and surface area of the wedge *ABCDEF* in which

Angle ABC = 90°
*AB* = 5 cm, *AC* = 13 cm
*BC* = 12 cm and *CD* 20 cm




**16.** *ABC* is a right angled triangle. *AB* is length of 4 m and *BC* is of length 13 m.

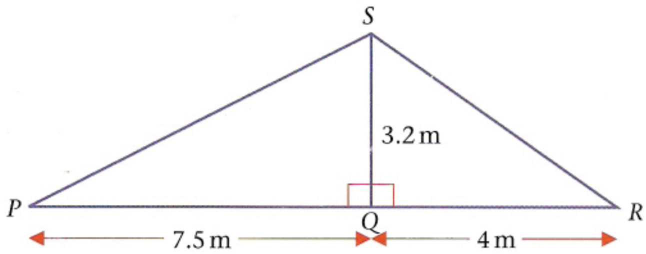
**a)** Calculate the length of *AC*. (Use $153^{1/2} = 12.37$)
**b)** Calculate the size of Angle *ABC*.




**17.** The diagram represents the frame, *PQRS,* of a roof.

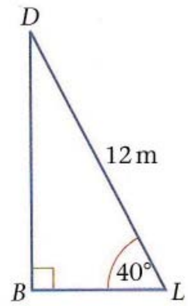
*PQ* = 7.5 m, *QR* = 4 m, *SQ* = 3.2 m

**a)** Calculate the length of *PS*. (Use $66.49^{1/2} = 8.15$
**b)** Calculate the size of the angle SRQ.



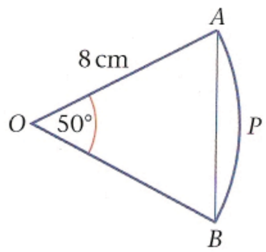

**18.** The diagram shows a ladder *LD* of length 12 m resting against a vertical wall. The ladder makes an angle of 40° with the horizontal.

Calculate the distance *BD* from the base of the wall to the top of the ladder.




**19.** Here is a sector of a circle, *OAPB*, with centre *O*.

Calculate
**a)** The length of the chord AB
**b)** The length of the arc APB
**c)** The area of the sector OAPB
**d)** The area of the segment APB



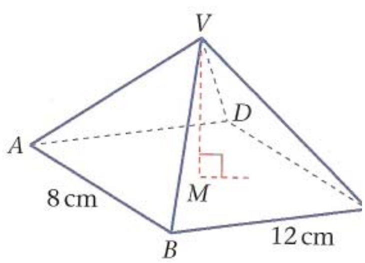

**20.** Calculate the volume and surface area of a cylinder with circular base of radius 12 cm and with vertical height 20 cm.

**21.** Calculate the volume of a sphere of diameter 8 cm.

**22.** VABCD is a pyramid with a rectangular base ABCD.

*AB* = 8 cm, *BC* = 12 cm
*M* is at the centre of *ABCD* and *VM* = 35 cm.
*V* is vertically above *M*.

Calculate the volume of *VABCD*.




**23.** A cone has a circular base of radius 20 cm.
The slant height of the cone is 30 cm. Calculate the volume of the cone.